THE STRESS ANALYSIS OF AN ICEBREAKER BOW

by

Richard L. DeVries

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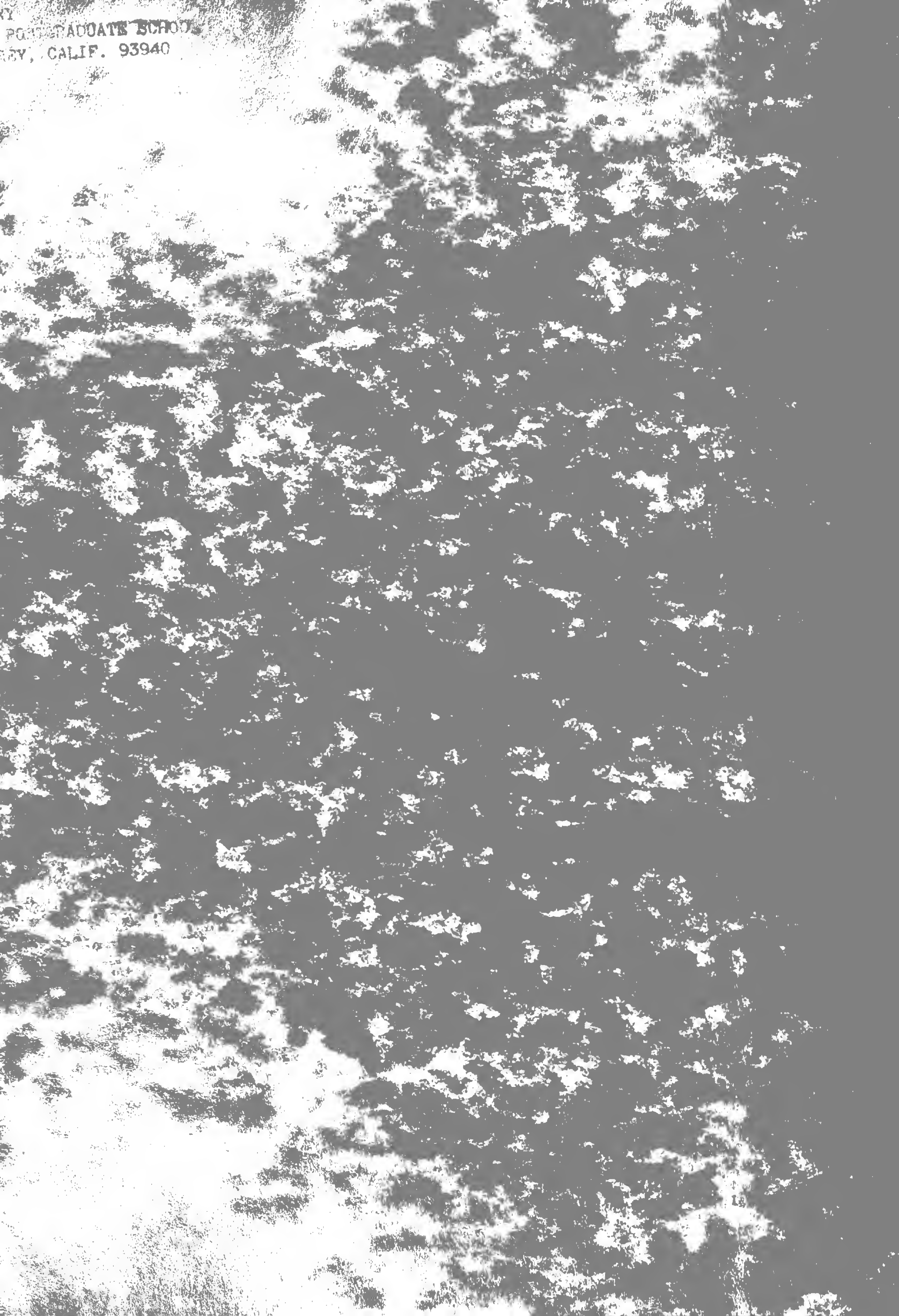

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ABSTRACT

The topic of this paper is the stress analysis of an icebreaker bow. The purpose in writing this paper was to familiarize myself with the various methods that have been developed in determining bow loads in ice breaking and the methods that have been developed to compute the stresses in a ship's structure. From a study of many methods, I wanted to select one that would be most useful to me in my day to day work when I depart from M.I.T. I selected as the ship to model the United States Coast Guard Icebreaker WESTWIND.

In the first section, the various methods of determining bow loads in ice breaking was studied. The various methods that were put forth by scholars and engineers over the past half-century gave bow loads ranging from one hundred and fifty-four tons to values as high as twenty-nine hundred and fifty tons of force at the bow in a vertical direction. I chose as the best results to use in loading the bow, those values which are determinable from the computer program developed by Dr. White in the doctoral thesis, "The Dynamically Developed Force at the Bow of an Icebreaker." The computer program developed by Dr. White gives the vertical and horizontal forces acting on the bow as function of the length between perpendiculars, the beam at the waterline, mean draft, displacement, bow angle, spread angle complement, impact velocity, water plane coefficient, longitudinal position of the center of floatation, the longitudinal position of the center of gravity, the height of the center of gravity, height of the thrust line above the baseline, bollard thrust that would be obtained for RPM being used during the crushing phase of the ice, longitudinal metercentric height, kenetic friction coefficient of ice and the compressive failure stress of ice. The vertical load determined by his computer program for the WESTWIND ramming solid ice at fifteen knots gave a peak value of two thousand tons.

The next section of this paper covers the various methods available for determining the stress loading inside the structure. The most used computer programs that were reported upon were the STRESS and FRAN programs. Both of these programs being strictly a frame analysis of the structure leave much to be desired in the complete knowledge of the stresses present in a complicated structure such as an icebreaker.

The most promising and most exact analysis that is available at the present time, utilizes the finite element method. A program developed and presently being revised by NASA that quickly and efficiently analyzes almost any structure is the "Structural Analysis and Matrix Interpretive System" (SAMIS). The remainder of the paper gives a short summary of the program, the data required by the analyst, and finally the necessary steps required for the analysis of the icebreaker WESTWIND.

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ACKNOWLEDGEMENTS

Much credit for the completion of this thesis must go to Professor Alaa E. Mansour for his ever willing assistance. I would also like to thank LCDR Melburg for his assistance and guidance over these past two semesters.

Words cannot express the thanks for the patience my wife and family had with me while blueprints and books cluttered every corner of our house.

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INTRODUCTION

In the Shipbuilder by John Ruskin, 1819-1900, he states, "Take it all in all a ship of the line is a most honorable thing that man as a gregarious animal has ever produced. Into that he has put as much of his human patience, common sense, forethought, experimental philosophy, self control, habits of order and obedience, thoroughly wrought handiwork, definance of brute elements, careless courage, careful patriotism, and calm expectation of the judgement of God as can be put into a space 300 feet long and forty feet broad," (Ref. 1)*

Since that time, many ships have made history--some of them by going to the bottom of the sea; others, in the defiance of the brute elements have trespassed the Arctic waste. This type of ship is what this thesis is all about.

The icebreaker hull at its birth was merely a standard hull with extra reinforcement in the bow. As time progressed, changes in design were based largely on the success and failure of past designs. Efficient ice breaking (or what was believed to be efficient ice breaking) led to the thirty degree angle bow. Removal of the unsuccessful

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*References are listed at the end of this report.

bow propeller on the wind class breaker produced the step which prevented the vessel from riding completely up on the ice. With the advent of the computer, more efficient and better designs have been produced.

Although very few papers have been written on the subject of icebreaker design, those that have been produced have made far reaching effects on the design of the icebreaker hull.

METHODS OF DETERMINING BOW LOADS

The earlier works were done by the Russians. M. K. Tarshis in his paper, "Ice Loads Acting on Ships," (Ref. 2) put forth a formula to determine the impact load on the vessel. This formula considers the speed, the angle of blow, and the square root of the relative mass and the relative rigidity of the ship. He uses as an example a 655 ton displacement vessel in contact with an ice floe that is 25 meters in diameter and one meter in thickness. From his formula he deduces that the impact load is 220 tons. He assumed in this work a crushing strength of ice of 570 pounds per square inch.

The general idea of this paper is that the speed of the vessel and the angle of blow in the area of contact with the ice floe are the major contributors to the impact load on the vessel; hence, if you double the speed of the vessel you double the impact load on the bow of the vessel.

"The paper has a high theoretical content and requires many parameters difficult to determine. The relative angles of the hull form at the point of impact are necessary and these can vary quite appreciably in a relatively short distance." (Ref. 2, pg. 5)

L. M. Nogid in his paper, "Impact of Ships with Ice," (Ref. 3) attempts to determine the reduction of speed of the icebreaker as it contacts the ice. An interesting sidelight of this paper is that the theoretical amount of force required to initiate the crack in the ice is quite small, but that the force required to propogate that crack might be quite large. It also goes on to show that the load required to break off pieces of the ice is more than three times the load required to initiate a crack.

In his paper, Mr. Nogid gives a method to determine the speed reduction a vessel will have when coming into contact with an ice floe of given diameter if you know the maximum load the ship could withstand at the point of impact and the angle formed with the ice at the point of contact. The area can also be determined if you know what the strength of the ice is in the area in which the ice breaking operation is being carried out.

He divides the forces and the strength requirements of the hull into two different areas. One is impact and the other is compression.

Under the impact force, he determines that this force is equal to the mass of the ship times the velocity of the ship, modified by parameters which are dependent upon the angle of the hull at the point of impact, the area of impact and other similar items.

Under compression, the author assumes that the loading is directly related to the crushing strength of the ice. He also suggests some relationships between the changes of length and the strength of the side frames. His formula suggests that the strength of side frames is related to the cube root of the ratio prototype to the proposed length. He also goes further in an attempt to relate hull strength to the frame spacing of the vessel. This paper, in general, is the theoretical approach to the old idea of taking a successful design and changing the dimensions to make a more capable design.

U. N. Raskin in his paper, "Method of Determining the Stresses in Decks and Transverse Bulkheads Caused by Ice Loads," (Ref. 4) uses a compressive ice load on the side framing of a ship to determine what the stresses are in the decks and bulkheads. He divides the decks into strips with the only load carrying area being the deck itself and those beams in direct contact with the deck. The stringer is considered to be in simple compression, while the remaining

strips act as rigid members to bending and carry only shear loads. In between these strips are imaginary elastic bands which allow relative movement back and forth between the bands. The bulkheads are considered as simple plates in end compression.

Using these models, he presents a method to determine stresses in these decks and bulkheads. I believe it would be rather difficult to accurately model a complicated structure, such as an icebreaker, in this form and achieve accurate results.

In September of 1965, R. M. White completed his doctoral thesis, "Dynamically Developed Force at the Bow of an Icebreaker." (Ref. 5) In this thesis, Dr. White developed a computer program for the prediction of the dynamically developed force under the bow of an icebreaker while ramming solid ice.

The solution is based primarily on Newton's Law of Motion. The problem was broken down into two basic phases. The crushing phase represents the local crushing of the ice to accommodate the bow. The sliding phase represents the sliding up of the bow without further crushing. "The final state represents (temporary) equilibrium when motion has stopped; the vertical force at the bow at this state is relatively sustained and is the most effective in breaking the ice." (Ref. 5, pg. 3)

Dr. White gives one of the earlier developed force equations which is attributed to R. Ruineberg in 1888 (Ref. 6).

$$F_{BZ} = \frac{T_{IB}(\cos i_B \cos \beta - f_k \sin i_B)}{(\sin i_B \cos \beta + f_k \cos i_B)}$$

where

F_{BZ} = Downward force

T_{IB} = Thrust available for ice breaking.

f_k = Coefficient of friction (suggested as .05).

β = The angle with respect to the center line plane of a normal to the shell.

i_B = The angle the stem makes with the base plane.

Dr. White states that Ruineberg assumed no momentum effects and no forward motion, so that all thrust is applied to ice breaking and is applied horizontally at all times. The direction of friction force remains the same during forward horizontal progress and the trim does not effect the solution. For the icebreaker WESTWIND, this would give a downward force of four hundred and fifty-four tons for this continuous mode of ice breaking.

The next equation Dr. White discussed was one developed by A. Kari in 1921 (Ref. 7).

$$F_{BZ} = \frac{4480 \ \Delta \ C \ L \sin \theta}{H}$$

where

Δ is displacement in tons

θ is change in trim

L is the length between perpendiculars

H is the draft in feet

and

$$C = \frac{BM_L \times H}{L^2} \simeq \frac{GM_L \times H}{L^2} = .07$$

where

BM_L is assumed to be equal to GM_L.

GM_L is longitudinal metacentric height in feet.

Kari makes the following assumptions: (1) no momentum effect, (2) the bow rises the thickness of the ice, (3) the distance from point of contact to the center of flotation and the center of gravity are the same, (4) effective displacement and the draft is not changed by load force, (5) the center of flotation and metacenter remain fixed, (6) C = .07, and (7) there is no frictional force. This would give a downward force of approximately seven hundred and fifty-eight for the icebreaker WESTWIND.

The third method discussed by Dr. White was one done by E. R. Simonson in 1936. (Ref. 8)

$$F_{BZ} = \frac{T_{IB}}{\tan\ (iB + \theta)}$$

In this method, Simonson assumed there are no momentum effects, friction is negligible, thrust is directed horizontally, the coefficient of friction serves as a pivot point, and there is no change in displacement. Dr. White states, "... Simonson's equation is limited to being a good approximation for the stopped equilibrium position." (Ref. 5, pg. 46) This would give a downward force of one hundred and seventy-three tons for the icebreaker WESTWIND.

The fourth work discussed by Dr. White on the downward force developed in ice breaking is one by L. V. Vinogradov which was published in 1946. (Ref. 9) Dr. White states that this is the first time that force due to ramming was put into useable mathematical form.

$$F_{BZ} = XT + \{X^2T^2 + \frac{Y}{A} W^2 \cdot \frac{V_o^2 \, [1 - (1 - e^2 \sin^2 i_B) - V_1^2]}{gD}\}^{**}$$

where

$$X = \frac{1 - \frac{f_k}{\cos \beta} \tan i_B}{1 + \frac{f_k}{\cos \beta} \cot i_B}$$

T = thrust in tons

$$Y = \frac{1}{1 + \frac{f_k}{\cos \beta} \cot i_B}$$

** Symbols have been changed to previously defined symbols.

ω = displacement

D = draft in ft.

$$A = \frac{C_B}{C_W} \left[1 + \frac{k_1}{k_2} \quad \frac{1}{4C_W} \right]$$

C_B = block coefficient

C_W = waterline coefficient

$k_1 = \frac{q}{L/2}$

q = distance from point of impact to center of flotation

$k_2 = \frac{1}{C_W L} \quad GM_L \cdot H \cdot C_B$

V_o = speed prior to impact

V_1 = speed while sliding up

Vinogradov assumed that thrust is always horizontal, change in trim and draft is small enough so as not to affect the waterplane characteristics and the metacentric height, and $GM_L = BM_L$.

In Vinogradov's equation the force downward is related to the frictional force, the angle of the bow, the spread angle of the bow, the coefficient of friction, the block coefficient, the waterline coefficient, the distance of impact from the center of flotation, the speed prior to impact, and the speed while sliding up. This would give a downward force of approximately two thousand nine hundred and sixty tons for the icebreaker WESTWIND.

The fifth equation discussed by Dr. White is one by F. Richardson in 1959 in some personal correspondence to Dr. White. "The development was almost identical to Vinogradov's but did modify some of his weaknesses to some extent. For example, Richardson uses a term for the loss of energy due to wave and frictional resistance (not ice) from the instant of contact up to the moment the ice breaks or motion ceases. He also recognizes an effective increase in the mass of the icebreaker due to entrained water." (Ref 5,p 49)

The final equation discussed by Dr. White was one developed by V. R. Milano in 1962 which is a modification of Vinogradov's equation. Dr. White states, "One of the main contributions was to express thrust as a function of 'Bollard Pull'." (Ref. 5, p. 51)

Dr. White in his analysis assumes:

1. The force normal to the plating is represented by the product of the area of contact and the compressive failure stress of the ice.
2. There is a friction force acting in the plane of the plating.
3. The icebreaker is treated as a "solid body."
4. The radius of gyration of an icebreaker can be assigned 0.266.

5. Crushing has ceased when velocity is in the direction of the angle of the bow plus the angle of trim.

6. During sliding phase, the point of contact is fixed and is at the level of the waterline.

Results show that the peak load occurs during the crushing phase.

In his computer program, the downward force is determined as a function of

		Computer Symbol
L	Length between perpendiculars, ft.	BP
B	Beam at waterline, ft.	B
H	Mean draft, ft.	H
Δ	Displacement, lbs.	DIS
i_B	Bow angle (from base line to stem), radians	BA
SA	Spread angle complement (normal to bow plating with respect to center line plane), radians	SA
V_1	Impace velocity, ft/sec.	V1
$\alpha = C_W$	α, Waterplane coefficient, dimensionless	AL
LCF	Longitudinal position of the center of flotation (- if aft of amidships, + if forward), ft.	CF

		Computer Symbol
CG	LCG, Longitudinal position of the center of gravity (- if aft of amidships, + if forward), ft.	CG
KG	Height of center of gravity above base line, ft.	GK
d	Height of thrust line above base line near center of gravity, ft.	D
T_{BOL}	Bollard thrust which would be obtained for rpm used during crushing and sliding, lbs.	TB
GM_L	Longitudinal metacentric height, ft.	GM
f_k	Kenetic coefficient of friction of ice/ship.	FK
σ	Compressive failure stress of ice, #/ft^2.	SIG

The following pages are Dr. White's complete program for developing this downward force. (Ref. 5, pp. 385-393).

```
*    M4045-3564,FMS,TEST,5,5,5000,0 DYNAMIC ICEBREAKING R.M.WHITE
*    XEQ
C    DYNAMIC ICEBREAKING  R.M. WHITE
36   READ 5,BP,B,H,DIS,BA,SA,V1,AL,CF,CG,GK,D,TB,GM,FK,FS,SIG
 5   FORMAT (4F15.3/4F15.3/4F15.3/4F15.3/F15.3)
     PRINT 41, BP,B,H,DIS,BA,SA,V1,AL,CF,CG,GK,D,TB,GM,FK,FS,SIG
41   FORMAT (6H  BP=,F15.3,FH  B=,F15.3,FH  H=,F15.3,7H  DIS=,F15.3
    1/6H  BA=,F15.3,6H  SA=,F15.3,6H  V1=,F15.3,6H  AL=,F15.3/
    26H  CF=,F15.3,6H  CG=,F15.3,6H  GK=,F15.3,5H  B=,F15.3/6H  TB
    3=,F15.3,6H  GM=,F15.3,6H  FK=,F15.3,6H  FS=,F15.3/7H  SIG=,
    4F15.3//)
     XM= (3.36E-2)*DIS
     ZM = (5,78E-2)*DIS
     RG = 0.22*BP
     THM = (5.0E-2)*(RG**2)*DIS
     DP = (1.76E-2)*DIS*BP**1.5
     DH = (5.29E-1)*DIS/BP**0.5
     TF = (64.2)*BP*B*AL
     SIBA = SINF(BA)
     COBA = COSF(BA)
     TABA = SIBA/COBA
     SISA = SINF(SA)
     COSA = COSF(SA)
     P1 = (SIG*TABA/SISA)*(SIBA*(COBA*(COSA+FK*SISA)-FK*SIBA)
     P3 = P2*(BP/2.-CG)+P1*(H-GK)
     A1 = THM
     B1 = BP
     C1 = DIS*GM
     D1 = V1**2*P3
     AL1 = -B1/(2.*A1)
     DISC1 = 4.*C1/A1-(B1**2)/(A1**2)
     IF (DISC1) 11,2,2
 2   BE1 = 0.5*SQRTF(DISC1)
     AA1 = (2.*D1/C1**2)*(A1-B1**2/C1)
     AA2 = (2.*D1/((C1**2)*BE1))*(B1-AL1*(A1-(B1**2)/C1))
     A2 = ZM
     B2 = DH
     C2 = TF
     D2 = -P2*V1**2
```

```
      AL2 = -B2(2.&A2)
      DISC2 = 4.*C2/A2-(B2**2)/(A2**2)
      IF (DISC2) 12,3,3
    3 BE2 = 0.5*SQRTF(DISC2)
      BB1 = (2.*D2/C2**2)*(A2-B2**2/C2)
      BB2 = (2.*D2/((C2**2)*BE2))*(B2-AL2*(A2-B2**2)/C2))
      PRINT 4, XM,ZM,RG,THM,DP,DH,TF,P1P2,A1,B1,C1,P3,D1,AL1,BE1,AA1,
     1AA2,A2,B2,C2,D2,AL2,BE2,BB1,BB2
    4 FORMAT (4E12.4/5E12.4/5E12.4/4E12.4/4E12.4/4E12.4//)
      T = -0.05
    1 T = T+0.05
      EAL1T + EXPF(AL1*T)
      COB1T = COSF(BE1*T)
      SIBIT = SINF(BE1*T)
      TH = EAL1T*(AAL*COB1T+AA2*SIB1T)+D1*(T**2)/C1-2.*B1*D1*T/(C1**2)
     1-AA1
      THD = AL1*EAL1T*(AA1*COB1T+AA2*SIB1T)+EAL1T*(-AA1*BE1*SIB1T+AA2
     1*BE1*COB1T)+2.*B1*T/C1-2.*B1*D1/C1**2
      THDD = (AL1**2-BE1**2)*EAL1T*(AA1*COB1T+AA2*SIB1T)+2.*AL1*BE1*
     1EAL1T*(-AA1*SIB1T+AA2*COB1T)+2.*D1/C1
      EAL2T = EXPF(AL2*T)
      COB2T = COSF(BE2*T)
      SIB2T = SINF(BE2*T)
      Z = EAL2T*(BB1*COB2T+B2*SIB2T)+D2*(T**2)/C2-2.*B2*D2*T/(C2**2)-BB1
      ZD = AL2*EAL2T*(BB1*COB2T+BB2*SIB2T)+EAL2T*(-BB1*BE2*SIB2T+BB2
     1*BE2*COB2T)+2.*D2*T/C2-2.*B2*D2/C2**2
      ZDD = (AL2**2-BE2**2)*EAL2T*(BB1*COB2T+BB2*SIB2T)+2.*AL2*BE2*
     1EAL2T*(-BB1*SIB2T+BB2*Cob2T)+2.*D2/C2
      X = V1*(T-P1*T**3/(12.*XM))
      XD = SQRTF(V1**2-2.*P1*X**3/(3.*XM))
      XDD = -P1*X**2/XM
      FXC = P1*X**2
      FZC = P2*X**2
      GAX = (BP/2.-CG)-((H-GK)+(BP/2.-CG)/TABA)*TH+Z/TABA
      GAZ = H-GK+Z
      TAGA = (GAX*THD-ZD)/(XD-GAZ*THD)
      DIF = SINF(BA+TH)/COSF(BA+TH)-TAGA
      PRINT 6,T,TH,THD,THDD,Z,ZD,ZDD,X,XD,XDD,FXC,FZC,TAGA,GAX,GAZ,GIF
```

```
 6   FORMAT (F11.5/3F11.5/3F11.5/3F11.5/2F12.5/4F11.5//)
     IF (XD) 38,38,37
38   PRINT 39, FZC
     GO TO 36
39   FORMAT (44H   SHIP STOPPED DURING CRUSHING PHASE, FZC2=,E12.5//)
37   IF (DIF)  14,14,7
 7   TL = T
     THL = TH
     THDL = THD
     THDDL = THDD
     ZL = Z
     ZDL = ZD
     ZDDL = ZDD
     XL= X
     XDL = XD
     XDDL = XDD
     FXCL = FXC
     FZCL = FZC
     TAGAL = TAGA
     GAXL = GAX
     GAZL = GAZ
     DIFL = DIF
     GO TO 1
14   TERP = DIFL/(DIFL-DIF)
     T2 = TL+TERP*(T-TL)
     TH2 = THL+TERP*(TH-THL)
     THD2 = THDL+TERP*(THD-THDL)
     THDD2 = THDDL+TERP*(THDD-THDDL)
     Z2 = ZL+TERP*(Z-ZL)
     ZD2 = ZDL+TERP*(ZD-ZDL)
     ZDD2 = ZDDL+TERP*(ZDD-ZDDL)
     X2 = XL+TERP*X-XL)
     XD2 = XDL+TERP*(XD-XDL)
     XDD2 = XDDL+TERP*(XDD-XDDL)
     FXC2 = FXCL+TERP*(FXC-FXCL)
     FZC2 = FZCL+TERP*(FZC-FZCL)
     TAGA2 = TAGAL+TERP*(TAGA-TAGAL)
```

```
      GAX2 = GAXL+TERP*(GAX-GAXL)
      GAZ2 = GAZL+TERP*(GAX-GAZL)
      DIF2 = DIFL+TERP*(DIF-DIFL)
   10 PRINT 15, T2,TH2,THD2,THDD2,Z2,ZD2,ZDD2,X2,XD2,XDD2,FXC2,FZC2,
     1TAGA2,GAX2,GAZ2,DIF2
   15 FORMAT (6H   T2=,F11.5/7H   TH2=,F11.5,8H   THD2=,F11.5,
     19H   THDD2=,F11.5/6H   Z2=,F11.5,7H   ZD2=,F11.5,8H   ZDD2=,F11.5/
     26H   X2=,F11.5,7H   XD2=,F11.5,8H   XDD2=,F11.5/8H   FXC2=,E12.5,
     38H   FZC2=,E12.5/9H   TAGA2=,F11.5,8H   GAX2=,F11.5,8H   GAX2=,
     4F11.5,8H   DIF2=,F11.5//)
      GO TO 16
   11 PRINT 13,DISC1
   13 FORMAT (E12.4)
      GO TO 36
   12 PRINT 13,DISC2
      GO TO 36
C     ICEBREAKER SLIDING PHASE SOLUTION R. M. WHITE
   16 AS = COSA*SIBA+FK*COBA
      BS = COSA*COBA-FK*SIBA
      XDD2 = -1.0
      ZDD2 = 0.0
      THDD2 = 0.0
      P4 = GAX2+X2
      P5 = 1.+(AS/BS)**2
      HGK = H-GK
      CGCF = CG-CF
      GKD = GK-D
      A11 = -XM
      B11 = -(TB/V1)*(1.+AS*TH2/BS+P5*TH2**2)
      C11 = 0.0
      A12 = ZM*(AS/BS+P5*TH2)
      B12 = DH*(AS/BS+P5*TH2)
      C12 = TF*(AS/BS+P5*TH2)
      A13 = 0.0
      B13 = 0.0
      C13 = TB*(AS/BS-AS*XD2/(BS*V1)+2.*P5*TH2-2.*P5*XD2*TH2/V1)
     1+TF*(AS*CGCF/BS+P5*Z2+2.*P5*CGCF*TH2)+P5*DH*ZD2+P5*ZM*ZDD2
      D1 = -TB*(1.+AS*XD2*TH2/(BS*V1)-P5*TH2**2+2.*P5*XD2*TH2**2/V1)
     1+TF*(P5*Z2*TH2+P5*CGCF*TH2**2)+P5*DH*ZD2*TH2+P5*ZM*ZDD2*TH2
```

```
 A21 = 0.0
 B21 = TB*(P4*TH2/V1-TH2*X2/V1+AS*HGK*TH2/(BS*V1)+P5*HGK*TH2**2/V1
1+AS*TH2*Z2/(BS*V1)+P5*Z2*TH2**2/V1-GKD/V1)
 C21 = TB*(TH2-XD2*TH2/V1)+TF*(Z2+CGCF*TH2)+DH*ZD2+ZM*ZDD2
 A22 = ZM*(-P4+X2-AS*HGK/BS-P5*HGK*TH2-AS*Z2/BS-P5*TH2*Z2)
 B22 = DH*(-P4+X2-AS*HGK/BS-P5*HGK*TH2-AS*Z2/BS-P5*TH2*Z2)
 C22 = TF*(X2-AS*HGK/BS-P5*HGK*TH2-P4-2.*AS*Z2/BS-AS*CGCF*TH2/BS
1-2.*P5*Z2*TH2-P5*CGCF*TH2**2-GM*TH2)+TB*(-AS*TH2/BS+AS*XD2*TH2/
2(BS*V1)-P5*TH2**2+P5*XD2*TH2**2/V1)-DH*(-AS*ZD2/BS-P5*ZD2*TH2)
3+ZM*(-AS*ZDD2/BS-P5*ZDD2*TH2)
 A23 = -THM
 B23 = -DP
 C23 = TB*(-P4+P4*XD2/V1+X2-XD2*X2/V1-AS*HGK/BS+AS*HGK*XD2/(BS*V1)
1-2.*P5*HGK*TH2+2.*P5*HGK*XD2*TH2/V1-AS*Z2/BS+AS*XD2*Z2/(BS*V1)
2-2.*P5*Z2*TH2+2.*P5*XD2*TH2*Z2/V1)+TF*(-P4*CGCF+CGCF*X2-AS*CGCF*
3HGK/BS-P5*HGK*Z2-2.*P5*CGCF*HGK*TH2-AS*CGCF*Z2/BS-P5*Z2**2-2.*P5
4*CGCF*PH2Z2-GM*Z2-2.*GM*CGCF*TH2)+DH*(-P5*HGK*ZD2-P5*ZD2*Z2)+ZM*
5(-P5*HGK*ZDD2-P5*ZDD2*Z2)-DIS*GM
 D2 = TB*(P4*XD2*TH2/V1+TH2*X2-2.*XD2*TH2*X2/V1+AS*HGK*XD2*TH2/(BS*
1V1)-P5*HGK*TH2**2+2.*P5*HGK*XD2*PH2**2/V1-AS*TH2*Z2/BS+2.*AS*XD2
2*TH2*Z2/(BS*V1)-2.*P5*PH2**2*Z2+3.*P5*ZD2*Z2*TH2**2/V1-GKD)+TF*(
3Z2*X2+CGCF*TH2*X2-P5*HGK*Z2*TH2-P5*CGCF*HGK*TH2**2-AS*Z2**2/BS-AS
4*CGCF*TH2*Z2/BS-2.*P5*TH2*Z2**2-2.*P5*CGCF*Z2*TH2**2-GM*TH2*Z2-GM*
5CGCF*TH2**2)+DH*(ZD2*X2-P5*HGK*ZD2*TH2-AS*ZD2*Z2/BS-2.*P5*ZD2*TH2*
6Z2)+ZM*(ZDD2*X2-P5*HGK*ZDD2*TH2-AS*ZDD2*Z2/BS-2.*P5*ZDD2*TH2*Z2)
 A31 = 0.0
 B31 = 0.0
 C31 = 1.0
 A32 = 0.0
 B32 = 0.0
 C32 = 1./TABA-TH2/SIBA**2
 A33 = 0.0
 B33 = 0.0
 C33 = -GAX2/TABA+GAX2*TH2/SIBA**2+GAZ2
 D3 = X2-GAX2*TH2/TABA+ GAX2*TH2**2/SIBA**2+Z2/TABA-Z2*TH2/SIBA**2
1+GAZ2*TH2
 PRINT 17,AS,BS,P4,P5,A11,B11,C11,A12,B12,C12,A13,B13,C13,D1,
1A21,B21,C21,A22,B22,C22,A23,B23,C23,D2,A31,B31,C31,A32,B32,C32,
```

```
      2A33,B33,C33,D3
   17 FORMAT (4E14.6//3E14.6/3E14.6/3E14.6/E14.6//3E14.6/3E14.6/3E14.6/
     1E14.6//3E14.6/3E14.6/3E14.6/E14.6//)
      D11 = A11*XD2+D11*X2+A12*ZD2+B12*Z2
      D12 = A11*X2+A12*Z2
      D13 = D1
      D21 = B21*X2+A22*ZD2+B22*Z2+A23*THD2+B23*TH2
      D22 = A22*Z2+A23*TH2
      D23 = D2
      D33 = D3
      DD4 = A11*A22*C33+A12*A23*C31-A11*A23*C32
      DD3 = A11*B22*C33+B11*A22*C33+A12*B23*C31+B12*A23*C31-A11*B23*C32
     1-B11*A23*C32-A12*B21*C33
      DD2 = A11*C22*C33+B11*B22*C33+A12*C23*C31+B12*B23*C31+C12*A23*C31
     1-A11*C23*C32-B11*B23*C32-A12*C21*C33-B12*B21*C33-C13*A22*C31
      DD1 = B11*C22*C33+B12*C23*C31+C12*B23*C31+C13*B21*C32-B11*C23*C32
     1-B12*C21*C33-C12*B21*C33-C13*B22*C31
      DD0 = C12*C23*C31+C13*C21*C32-C12*C21*C33-C13*C22*C31
      U13 = D12*A22*C33+A12*A23*D33-D12*A23*C32-A12*B22*C33
      U12 = D11*A22*C33+D12*B22*C33+A12*B23*B33+B12*A23*D33-D11*A23*C32
     1-D12*B23*C32-A12*D21*C33-B12*D22*C33
      U11 = D11*B22*C33+D12*C22*C33+D13*A22*C33+A12*C23*D33+B12*B23*D33
     1+C12*A23*D33+C13*D22*C32-D11*B23*C32-D12*C23*C32-D13*A23*C32
     2-A12*A23*C33-B12*D21*C33-C12*D22*C33-C13*A22*D33
      U10 = D11*C22*C33+D13*B22*C33+B12*C23*D33+C12*B23*D33+C13*D21*C32
     1-D11*C23*C32-D13*B23*C32-B21*D23*C33-C12*D21*C33-C13*B22*B33
      U09 = D13*C22*C33+C12*C23*D33+C13*D23*C32-D13*C23*C32-C12*D23*C33
     1-C13*C22*D33
      U23 = A11*D22*C33+D12*A23*C31-A11*A23*D33
      U22 = A11*D21*C33+B11*D22*C33+D11*A23*C31+D12*B23*C31-A11*B23*D33
     1-B11*A23*D33-D12*B21*C33
      U21 = A11*D23*C33+B11*D21*C33+D11*B23*C31+D12*C23*C31+D13*A23*C31
     1-A11*C23*D33-B11*B23*D33-D11*B21*C33-D12*C21*C33-C13*D22*C31
      U20 = B11*D23*C33+D11*C23*C31+D13*B23*C31+C13*B21*D33-B11*C23*D33
     1-D11*C21*C33-D13*B21*C33-C13*D21*C31
      U19 = D13*C23*C31+C13*C21*D33-D13*C21*C33-C13*D23*C31
      U33 = A11*A22*D33+A12*D22*C31-A11*D22*C32-D12*A22*C31
```

```
      U32 = A11*B22*D33+B11*A22*D33+A12*D21*C31+B12*D22*C31+D12*B21*C32
     1-A11*D21*C32-B11*D22*C32-A12*B21*D33-D11*A22*C31-D12*B22*C31
      U31 = A11*C22*D33+B11*B22*D33+A12*D23+C31+B12*D21*C31+C12*D22*C31
     1+D11*B21*C32+D12*C21*C32-A11*D23*C32-B11*D21*C32-A12*C21*D33
     2-B12*B21*D33-D11*B22*C31-D12*C22*C31-D13*A22*C31
      U30 = B11*C22*D33+B12*D23*C31+C12*D21*C31+D11*C21*C32+D13*B21*C32
     1-B11*D23*C32-B12*C21*D33-C12*B21*D33-D11*C22*C31-D13*B22*C31
      U29 = C12*D23*C31+D13*C21*C32-C12*C21*D33-D13*C22*C31
      PRINT 13,D11,D12,D13,D21,D22,D23,D33,DD4,DD3,DD2,DD1,DD0,U13,U12,
     1U11,U10,U09,U23,U22,U21,U20,U19,U33,U32,U31,U30,U29
      WB4 = DD3/DD4
      WB3 = DD2/DD4
      WB2 = DD1/DD4
      WB1 = DD0/DD4
      W6 = -64.*WB4**6
      W5 = 96.*WB4**6
      W4 = (WB4**4)*(-48.*WB4**2-32.*WB3)
      W3 = (WB4**3)*(32.*WB3*WB4+8.*WB4**3)
      W2 = (WB4**2)*(16.*WB1-4.*WB3**2-4.*WB2*WB4-8.*WB3*WB4**2)
      W1 = (WB4)*(-8.*WB1*WB4+2.*WB3**2*WB4+2.*WB2*WB4**2)
      W0 = WB1*WB4**2-WB2*WB3*WB4+WB2**2
      C = 0
   19 CL = C
      TOTL = TOT
      C = C+0.001
      TOT = W6*C**6+W5*C**5+W4*C**4+W3*C**3+W2*C**2+W1*C+W0
      PRINT 13, C,TOT
      IF (TOT) 19,20,20
   20 C = CL-TOTL*0.001/(TOT-TOTL)
      PRINT 13, C
      AL3 = C*WB4
      DISC3 = (-WB2-3.*WB4*(AL3**2)+4.*(AL3**3)+2.*WB3*AL3)/(4.*AL3-WB4)
      IF (DISC3) 21,22,22
   21 PRINT 13, DISC3
      GO TO 36
   22 BE3 = SQRTF (DISC3)
      AL4 = (1.-2.*C)*WB4/2.
```

```
      DISC4 = WB3-(BE3**2)-(AL3**2)-4.*AL3*AL4-(AL4**2)
      IF (DISC4) 23,24,24
   23 PRINT 13, DISC4
      GO TO 36
   24 BE4 = SQRTF (DISC4)
      G3 = AL3**2+BE3**2
      G4 = AL4**2+BE4**2
      PRINT 13, AL3,BE3,AL4,BE4,G3,G4
      A4X = U13/DD4
      A3X = U12/DD4
      A2X = U11/DD4
      A1X = U10/DD4
      A0X = U09/DD4
      PRINT 13, A4X,A3X,A2X,A1X,A0X
      C1X = A0X/(G3*G4)
      D1X = A4X-C1X
      D2X = A3X-C1X*WB4
      D3X = A2X-C1X*WB3
      D4X = A1X-C1X*WB2
      C6X = (2.*(AL3-AL4)*(G4*D3X-D1X*G4**2-2.*AL4*D4X)+(G3-G4)*(-G4*D2X
     1+2.*AL4*G4*D1X+D4X))/(2.*(AL3-AL4)*(2.*AL3*G4-2.*AL4*G3)+(G3-G4)
     2**2)
      C5X = (D4X-G3*C6X)/G4
      C4X = (G4*(D2X-2.*AL4*D1X)-D4X+C6X*(G3-G4))/(2.*G4*(AL3-AL4))
      C3X = D1X-C4X
      P23X = BE3*C3X
      P13X = C5X-AL3*C3X
      P24X = BE4*C4X
      P14X = C6X-AL4*C4X
      PRINT 13, C1X,D1X,D2X,D3X,D4X,C6X,C5X,C4X,C3X,P23X,P13X,P24X,P14X
      A4Z = U23/DD4
      A3Z = U22/DD4
      A2Z = U21/DD4
      A1Z = U20/DD4
      A0Z = U19/DD4
      PRINT 13,A4Z,A3Z,A2Z,A1Z,A0Z
      C1Z = A0Z/(G3*G4)
```

```
 D1Z = A4Z-C1Z
 D2Z = A3Z-C1Z*WB4
 D3Z = A2Z-C1Z*WB3
 D4Z = A1Z-C1Z*WB2
 C6Z = (2.*(AL3-AL4)*(G4*D3Z-D1Z*G4**2-2.*AL4*D4Z)+(G3-G4)*(-G4*D2Z
1+2.*AL4*G4*D1Z+D4Z))/(2.*(AL3-AL4)*(2.*AL3*G4-2.*AL4*G3)+(G3-G4)
2**2)
 C5Z = (D4Z-G3*C6Z)/G4
 C4Z = (G4*(D2Z-2.*AL4*D1Z)-D4Z+C6Z*(G3-G4))/(2.*G4*(AL3-AL4))
 C3Z = D1Z-C4Z
 P23Z = BE3*C3Z
 P13Z = C5Z-AL3*C3Z
 P24Z = BE4*C4Z
 P14Z = C6Z-AL4*C4Z
 PRINT 13, C1Z,D1Z,D2Z,D3Z,D4Z,C6Z,C5Z,C4Z,C3Z,P23Z,P13Z,P24Z,P14Z
 A4T = U33/DD4
 A3T = U32/DD4
 A2T = U31/DD4
 A1t = U30/DD4
 A0T = U29/DD4
 PRINT 13, A4T,A3T,A2T,A1T,A0T
 C1T = A0T/(G3*G4)
 D1T = A4T-C1T
 D2T = A3T-C1T*WB4
 D3T = A2T-C1T*WB3
 D4T = A1T-C1T*WB2
 C6T = (2.*(AL3-AL4)*(G4*D3T-D1T*G4**2-2.*AL4*D4T)+(G3-G4)*(-G4*D2T
1+2.*AL4*G4*D1T+D4T))/(2.*(AL3-AL4)*(2.*AL3*G4-2.*AL4*G3)+(G3-G4)
2**2)
 C5T = (D4T-G3*C6T)/G4
 C4T = (G4*(D2T-2.*AL4*D1T)-D4T+C6T*(G3-G4))/(2.*G4*(AL3-AL4))
 C3T = D1T-C4T
 P23T = BE3*C3T
 P13T = C5T-AL3*C3T
 P24T = BE4*C4T
 P14T = C6T-AL4*C4T
 PRINT 13, C1T,D1T,D2T,D3T,D4T,C6T,C5T,C4T,C3T,P23T,P13T,P24T,P14T
```

```
      T = -0.100
   25 T = T+0.100
   27 EAL3T = EXPF (AL3*T)
      COB3T = COSF (BE3*T)
      SIB3T = SINF (BE3*T)
      EAL4T = EXPF (AL4*T)
      COB4T = COSF (BE4*T)
      SIB4T = SINF (BE4*T)
      X = C1X+(1./(BE3*EAL3T))*(P23XCOB3T+P13X*SIB3T)+(1./(BE4*EAL4T))
     1(P24X*COB4T+P14X*SIB4T)
      XD = (-AL3/(BE3*EAL3T))*(B23X*COB3T+P13X*SIB3I)+(1./EAL3T)*(-P23X
     1*SIB3T+P13X*COB3T)-(AL4/(BE4*EAL4T))*(P24X*COB4T+P14X*SIB4T)
     2+(1./EAL4T)*(-P24X*SIB4T+P14X*COB4T)
      XDD = ((AL3**2-BE3**2)/(BE3*EAL3T))*(P23X*COB3T+T13X*SIB3T)-(2.*
     1AL3/EAL3T)*(-P23X*SIB3T+P13X*COB3T)+((AL4**2-BE4**2)/(BE4*EAL4T))*
     2(P24X*COB4T+P14X*SIB4T)-(2.*AL4/EAL4T)*(-P24X*SIB4T+P14X*COB4T)
      Z = C1Z+(1./(BE3*EAL3T))*(P23Z*COB3T+P13Z*SIB3T)+(1./(BE4*EAL4T))*
     1(P24Z*COB4T+P14Z*SIB4T)
      ZD = (-AL3/(BE3*EAL3T))*(P23Z*COB3T+P13Z*SIB3T)+(1./EAL3T)*(-P23Z
     1*SIB3T+P13Z*COB3T)-(AL4/(BE4*EAL4T))*(P24Z*COB4T+P14Z*SIB4T)
     2+(1./EAL4T)*(-P24Z*SIB4T+P14Z*COB4T)
      ZDD = ((AL3**2-BE3**2)/(BE3*EAL3T))*(P23Z*COB3T+P13Z*SIB3T)-(2.*
     1AL3/EAL3T)*(-P23Z*SIB3T+P13Z*COB3T)+((AL4**2-BE4**2)/(BE4*EAL4T))*
     2(P24Z*COB4T+P14Z*SIB4T)-(2.*AL4/EAL4T)*(-P24Z*SIB4T+P14Z*COB4T)
      TH= C1T+(1./(BE3*EAL3T))*(B23T*COB3T+P13T*SIB3T)+(1./(BE4*EAL4T))*
     1(P24T*COB4T+P14T*SIB4T)
      THD= (-AL3/(BE3*EAL3T))*(P23T*COB3T+P13T*SIB3T)*(1./EAL3T)*(-P23T
     1*SIB3T+P13T*COB3T)-(AL4/(BE4*EAL4T))*(P24T*COB4T+P14T*SIB4T)
     2+(1./EAL4T)*(-P24T*SIB4T+P14T*COB4T)
      THDD= ((AL3**2-BE3**2)/(BE3*EAL3T))*(P23T*COB3T+P13T*SIB3T)-(2.*
     1AL3/EAL3T)*(-P23T*SIB3T+P13T*COB3T)+((AL4**2-BE4**2)/(BE4*EAL4T))*
     2(P24T*COB4T+P14T*SIB4T)-(2.*AL4/EAL4T)*(-P24T*SIB4T+P14T*COB4T)
      FBZS = -TB*TH+TB*XD*TH/V1-TF*Z-TF*CGCF*TH-DH*ZD-XM*ZDD
      WRAT = FBZS/(V1*DIS)
      VAX = XD-(HGK+Z)*THD
      TT = T+T2
      PRINT 26, TT,T,X,XD,XDD,Z,ZD,ZDD,TH,THD,THDD,FBZS,WRAT,VAX
```

```
26   FORMAT (14H  TOTAL TIME=,F11.5,5H  T=,F11.5/5H  X=,F11.5,
    16H  XD=,F11.5,7H  XDD=,F11.5/5H  Z=,F11.5,6H  ZD=,F11.5,
    27H  ZDD=,F11.5/6H  TH=,F11.5,7H  THD=,F11.5,8H  THDD=,F11.5/
    38H  FBZS=,E12.5,8H  WRAT=,F10.6,7H  VAX=,F11.5//)
     TEST1 = A11*XDD+B11*XD+A12*ZDD+B12*ZD+C12*Z+C13*TH-D13
     TEST2 = B21*ZD+C21*X+A22*ZDD+B22*ZD+C22*Z+A23*THD+B23*THD+C23*TH-
    1D23
     TEST3 = C31*X+C32*Z+C33*TH-D33
     PRINT 13, TEST1,TEST2,TEST3
     IF (VAX) 30,30,31
30   IF (VAX+0.02) 29,28,28
29   T = T-0.005
     GO TO 27
31   IF (VAX-0.02) 28,28,25
28   TT3 = TT
     T3 = T
     X3 = X
     XD3 = XD
     XDD3 = XDD
     Z3 = Z
     ZD3 = ZD
     ZDD3 = ZDD
     TH3 = TH
     THD3 = THD
     THDD3 = THDD
     FBZ3 = FBSZ
     WRAT3 = WRAT
     VAX3 = VAX
     PRINT 32, TT3,T3,X3,XD3,XDD3,Z3,ZD3,ZDD3,TH3,THD3,THDD3,FBZ3,
    1WRAT3,VAX3
32   FORMAT (17H  STATE 3 VALUES/14H  TOTAL TIME=,F11.5,6H  T3=,
    1F11.5/6H  X3=,F11.5,7H  XD3=,F11.5,8H  XDD3=,F11.5/6H  Z3=,
    2F11.5,7H  ZD3=,F11.5,8H  ZDD3=,F11.5/7H  TH3=,F11.5,8H  THD3=,
    3F11.5,9H  THDD3=,F11.5/8H  FBZ3=,E12.5,9H  WRAT3=,F10.6,
    48H  VAX3=,F11.5//)
     PRAT = FCZ2/FBZ3
```

```
      IF (PRAT-1.0) 42,44,44
42    PRINT 43, PRAT
43    FORMAT (46H   CAUTION, CRUSHING FORCE / SLIDING FORCE IS   ,F8.5//)
44    GAX3 = P4-X3
      GAZ3 = HGK+Z3
      Q1 = CGCF+GAX3
      A4 = GAZ3/(Q1*TF) GM*CGCF/(TF*Q1**2)-GM*GAX3/(TF*Q1**2)
      B4 = GAX3+GAZ3*TH3+GAZ3*Z3/Q1-GAZ3*GAX3*TH3/Q1+DIS*GM/(Q1*TF)
     1+GM*Z3/Q1-GM*GAX3*TH3/Q1-2.*GM*CGCF*Z3/Q1**2+2.*GM*CGCF*GAX3*TH3/
     2Q1**2-2.*GM*GAX3*Z3/Q1**2+2.*GM*GAX3**2*TH3/Q1**2
      C4 = DIS*GM*Z3/Q1-DIS*GM*GAX3*TH3/Q1+TF*GM*Z3**2/Q1-TF*GM*Z3*GAX3*
     1TH3/Q1-TF*GM*GAX3*TH3*Z3/Q1+TF*GM*GAX3**2*TH3**2/Q1-(TF*GM/Q1**2)*
     2(CGCF*Z3**2+CGCF*GAX3**2-2.*CGCF*GAX3*Z3*TH3+GAX3*Z3**2+
     3GAX3**3*TH3**2-2.*GAX3**2*Z3*TH3)
      PRINT 13,GAX3,GAZ3,Q1,A4,B4,C4
      DISC5 = (B4**2)-4.*A4*C4
      IF (DISC5) 34,33,33
34    PRINT 13, DISC5
      GO TO 36
33    RAD = SQRTF (DISC5)
      FBZ4 = (-B4+RAD)/(2.*A4)
      WRAT4 = FBZ4/(V1*DIS)
      TH4 = -FBZ4/(Q1*TF-Z3/Q1+GAX3*TH3/Q1
      Z4 = Z3+GAX3*(TH4-TH3)
      X4 = X3
      PRINT 35, X4,Z4,TH4,FBZ4,WRAT4
35    FORMAT (17H   STATE 4 VALUES/6H  X4=,F11.5,6H  Z4=,F11.5,
     17H  TH4=,F11.5/26H  VERTICAL FORCE AT BOW =,E12.5/16H  WHITE RA
     2TIO =,F10.6//)
      COBA5 = COSF(BA+TH4)
      SIBA5 = SINF(BA+TH4)
      A7 = COSA*COBA5+FS*SIBA5
      B7 = -COSA*SIBA5+FS*COBA5
      ET = FBZ4/((A7/B7)*COSF(TH4)-SINF(TH4))
      RAT = ET/TB
      PRINT 40,ET,RAT
```

```
40  FORMAT (22H   EXTRACTING THRUST =,E12.5/50H  RATIO OF EXTRACTING
   1THRUST TO BOLLARD THRUST IS ,F15.3//)
    GO TO 36
    END
```

STRESS ANALYSIS OF SHIPS STRUCTURES

In the analysis of the Coast Guard icebreaker GLACIER, in Lloyd's Register of Shipping Research and Technical Advisory Services Report No. 5095 (Ref.10), the IBM program STRESS was utilized. This is a computer program which performs a linear elastic analysis of a framed structure under the influence of a static load and is capable of solving two or three dimensional problems.

The condition of loading that was utilized was 400 psi ice pressure on the midship section of the icebreaker, and the ice condition was considered to be 10 feet in thickness. There were two conditions imposed with this 10 foot thickness of ice.

(1) Two feet of ice extending above the waterline and eight feet below it.

(2) Assume that all the ice was placed at the weakest point of the structure that was likely to encounter ice load in the ice breaking operation.

Once again in this program, you have a problem of the plating not carrying any load which certainly, in the analysis of an icebreaker where you have 2 1/2 and 3 inches of plating, will lead to large errors.

In the analysis that was carried out, two sets of end constraints were considered:

(1) Supports only at the extreme corners of the structure or to have four joints.

(2) Supports at all inboard ends of keelsons, etc. in addition to the condition above going a total of eight joints.

The bending moments obtained from this program with four and eight supports varied markedly with factors as low as two and with others as high as 60 and none of the variations showed any real relationships. The four support conditions gave values at various positions in the structure that at some points were much greater and sometimes much less than the corresponding points of the eight support conditions.

The results of the paper did not give any real concrete evidence as to what the actual stresses would be in the vessel considered under the 400 psi ice load condition. The conclusions were only that the structure should be of the grid structure instead of the truss structure. Once again, this is nothing more than a selection from the best of the successful designs to be applied to a new design.

Concurrent with the previous report, further investigation was also being done on these amidship frame areas by Paris Genalis. (Ref. 11) He also used the IBM computer program STRESS to analyze the amidship's section.

As he quotes in his report from

In his report, he breaks the analysis down into four stages:

(1) Consideration of part of only one frame.

(2) Consideration of one half of one frame.

(3) Two dimensional analysis of one complete ring frame.

(4) Three dimensional analysis of more than one complete ring frame.

Each of the analyses carried out was used to substantiate the assumptions and methods utilized in the next analysis. In the final three dimensional analysis, the plating was taken into account by replacing it with stiffeners to get away from the problems that Lloyd's of London had in their report.

Each of the problems considered in the three dimensional analysis, took a total of one hour or more of computer time. The number of calculations was very large, hence the probability of error was very large.

Mr. Genalis also went further in his analysis in that he considered machinery weight, steel weight, and the effects of varying amounts of buoyancy in addition to the ice loads. The wide variations in the results of his analysis point out the absolute need for proper modeling of the structure in order to determine realistic stresses.

Further limitations of the STRESS program is that it cannot handle curved members and thus assumes that each member is straight and slender.

Another application of the IBM program STRESS was completed by Lloyd's Register of Shipping, Research and Technical Advisory Service Department in Report No. 5051, (Ref. 12), in which a structure of shell plating subjected to ice pressure was analyzed. "The purpose of this investigation was to design a simple panel of shell plating stiffened in such a way so that the overall and local strength was sufficient to withstand a specified ice pressure uniformly distributed over the whole panel and for a concentrated load acting on one frame only for the full depth of the panel." (Ref. 12, p. 1)

Here again, we have results that do not take into account the shell plating other than to simulate it as part of the stiffeners. It does give results that compare well with simple beam theory, but does not give the information necessary for the designers who must, in these days of economy, depart from the large factor of safety of five or ten to insure that the vessel will be successful and safe from the environment it operates in.

The IBM STRESS program was also utilized by Consulter, Inc. of 1725 K Street, N.W., Washington, D. C. in "Polar Icebreaker Preliminary Structural Design and Special Studies," (Ref. 13) completed in August of 1968,

and gave a summary of known ice properties and an analysis of possible structures to be utilized in a new type of icebreaker called the M-10 at that time.

The ice loading used was that developed from the analysis by M. K. Tarshis, "Ice Loads Acting on Ships," (Ref. 14) a translation of a Russian text published in Rechnio Transport, Vol. 16, No. 12, 1957, pg. 19.

Using these values of ice loads they determined the capabilities of the proposed design along both elastic limits and plastic hinge limits. They included factors of 1 1/2 for impact loading and two for static condition loading to account for the lack of reliability of the values of the properties of ice.

The results once again lack aspects of reality due to the limitations of the STRESS program.

During recent years a new method called the finite element technique has been developed to give more accurate solutions for structure analysis. The first easily understood, comprehensive, presentation of the method was written by O. C. Zienkiewicz in his boom, "The Finite Element Method in Structural and Continuum Mechanics." (Ref. 15) Development of a computer program capable of analyzing a wide variety of structures was developed by the Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California, called the Structural Analysis and Matrix Interpretive System (SAMIS). (Ref. 16)

THE STRUCTURAL ANALYSIS AND MATRIX INTERPRETIVE SYSTEM

The Structural Analysis and Matrix Interpretive System (Ref. 16) uses elements that are at present restricted to flat triangular facets that are joined together along their edges, and line elements that are joined to the rest of the structure at their ends. All elements are capable of resisting stretching, shearing, bending and twisting loads. Heating, acceleration and pressure loads can also be analyzed. Additional loads can be introduced as energy equivalent concentrated loads at points on the structure.

Structural changes are defined by stresses, deflections, flexibilities and stiffnesses. It also has the capabilities of computing the natural frequencies of the structure and the mode shapes.

The finite element involved utilizes two ideas

1. The structure is divided into small elements
2. The problem is solved by a structural stiffness analysis.

Load deflection relations are taken for each element of the structure. The coefficients of these relations form the stiffness matrix. When the whole system is taken into account, the load deformation relations for the entire system stiffness matrix is developed by summing the stiffness matrices of the pieces composing the system. Where there are common grid points, the forces are simply added. Boundary conditions

are also formulated in matrix notation. The displacements at each of the corners of the facets and the ends of the beam that make up the complete structure are determined by solving systems of equations simultaneously. By taking these displacements, the stresses in each of the elements are determined.

In the Jet Propulsion Laboratory Technical Memorandum No. 33-317, (Ref. 17), the mathematical equations of the structure are given as:

$$[K]\{d\} - \lambda[K_i]\{d\} + [C][d] = \{P(t)\} - [M_s]\{d\}\{\sigma\} = [R]\{d\}$$

where $\{d\}$ = The vector of grid point displacements (dots indicate time derivatives).

$\{\sigma\}$ = The vector of element stresses.

$[K]$ = The small deflection stiffness matrix.

$[C]$ = The small deflection damping matrix.

$[K_i]$ = The initial stress stiffness matrix

λ = A scalar defining the magnitude of the initial stress distribution.

$[M_s]$ = The mass loading matrix.

$\{P\}$ = The force loading matrix.

$[R]$ = The matrix of stress coefficients.

Boundary conditions are applied on "d" and "λ" to make the above equations solvable.

The static and dynamic displacement response of the structure is given by the first equation and the stress associated with a given set of displacements is given by the second equation.

An assumed displacement function which is continuous along a facet's edge and continuous over the facet is used to develop the stiffness coefficients by minimizing the potential energy of the entire structure. The resulting stiffness coefficients define the forces and moments at the apexes of the triangle which satisfy macroscopic force and moment equilibrium conditions.

The displacements used to develop the stiffness coefficients are also used to develop the loading coefficients under the restraints of minimum potential energy.

The stress coefficients are developed using the same assumed displacements functions and stress-strain and strain deformation relationships. The stress so developed is the mean stress at the facet centroid.

Two coordinate systems are used to implement this program. The local system describes the grid points of each element as to their local relationship, and the common system describes the overall system. In the facet element, the local x-y plane is assumed to be coincident with the midplane of the facet. The displacements of each grid point are defined as a "d" vector which consists of translations in the x, y, and z directions, and the rotation about the x and y axis. Each component of these deflections is assumed to vary linearly

over the midplane surface. The deflection component at any point is therefore expressed as

$$d_i = a_i x + b_{iy} + c_i$$

In the stress-strain relation, it is assumed that the stress in the z direction is zero; assuming that the thickness radius ratio is small compared to one for the shell.

The selection of the triangular elements must be such that the stiffness matrix is positive definite. This can be done as long as the largest angle for any triangle is less than 90 degrees. If an angle is larger than 90 degrees in the facet, the stiffness matrix will be indefinite and thus cannot be used. The most accurate results are obtained if the triangles are all equilateral.

Discrete loads acting at a joint are described by forces acting in the x, y, and z directions and moments acting about the x and y axis. When torque is considered, it must be treated as a couple with forces in the x and y directions.

Local coordinate axis are used in developing the matrices of coefficient and, hence, the need for a transformation from the common axis. This is automatically accomplished if the grid points are given in the data as the common coordinates.

The line elements used are superposition of models for axial elongation, torsional rotation, shearing and bending.

Where stiffeners provide the resistance to bending, the classical bending element is used in the analysis. When using a facet, the shear bending element is used to provide the necessary stiffness from the supporting frames as this provides deformations which are consistent with those of the facet.

As in the facet element, assumed displacements are used in the development of the stiffness and loading coefficients. The stress coefficients are obtained directly from the stiffness matrix.

In the line element, the local x axis is considered to go down the centroid of the beam. The program automatically makes this transformation if the line element grid points are given in the common coordinates.

Due to the versatility of the program, the input necessary from the analysis for the SAMIS porgram is extensive. The first section of input for the program is the program control cards called "pusedo instructions." These cards control tape assignments, matrice maming and matrice manipulation. Card format is divided into alphanumeric names of the matrices, tape assignment instructions, instruction sequence number and program control information.

The card format is broken up into ten fields. The "O" field is the instruction sequence number. The "A", "B", and "C" fields contain the tape storage assignments,

the fields numbered 1, 2, and 3 contain matrix names, the "code" field contains the subroutine or subprogram name and the "E" field contains the control information for the operations to be performed. Card format is as follows:

F8.1	3X,I5	2X,A3,I3	3X,I5	2X,A3,I3	4X,A4	3X,I5	2X,A3,I3	2X,I6
0	A	1	B	2	CODE	C	3	E

Psuedo instructions are:

TABLE 1

(Ref. 16, p. 44)

OPERATION PSEUDO INSTRUCTIONS

Instructions	Interpretation
ADDS	Form C3 = A1 + B2
BILD	Construct small deflection stiffness, stress, loading, and/or mass matrices as A1, B1, C1, and F1, respectively.
CHIN	Form B2 such that B2 $B2^T$ = A1 and C3 = $B2^{-1}$ where A1 is symmetric and positive definite.
CHOL	Form C3 = $A1^{-1}$ B2 using Choleski decomposition.
CONT	Continuation Card (see pg 43)
CODE	Transform A1 to coded format as C3
COLS	Put the A1 matrix in column listing and call it C3.
DECO	Transform A1 to precoded format as C3.
FILL	Read A1, B2, and/or C3 into core.
FLIP	Form C3 = $A1^T$
INKS	Print matrices A1, B2, and/or C3
MULT	Form C3 = A1 B2
READ	Read matrices A1, B2, and/or C3 from cards.
ROWS	Put the A1 matrix in row listing and call it C3.
ROOT	Find latent roots and vectors of A1, a symmetric matrix. Let vectors be B2, roots C3.
SAVE	Write A1, B2, and/or C3 on tape.
SORT	Sort a matrix A1 by row or column as C3.
SUBS	Form C3 = A1 - B2
WASH	WASH A1 elements from B2 to produce C3.

Psuedo instructions called logic instructions provide the capability of loop operations and are also required for transfer of control to the success exit. When logic instructions are used, the "E" field indicates the number of times the logic instruction is to be carried out. Logic instructions are:

TABLE 2

(Ref. 16, p. 41)

LOGIC PSEUDO INSTRUCTIONS

Instructions	Interpretation
PREP	Prepare for multiple execution of the following instructions. Execute the instructions between the PREP and the next BACK instruction the number of times specified in the "E" field.
VARY	Vary matrix or tape numbers in the next instruction by augmenting corresponding field data by the specified integers after one pass.
BACK	Back up and repeat instructions after PREP.
ERRS	Disrupt errors can be corrected as follows.
SKIP	Skip the next "n" pseudo instructions where "n" is specified in the "E" field. (Skip cannot be included between a PREP and BACK instruction).
STOP	Stop this case and go to HALT.
PAWS	Pause in the calculations. Operator can restart at any time. (Not operative on IBM 7094-7040 DCS).
HALT	Halt and indicate a successful exit.

Control cards needed to direct computations for the most general case of static and pseudostatic loading and normal modes is a single analysis would be:

FORTRAN STATEMENT

0.0					CHEX				1
1.0	9001	WTR1	9002	ARC1	READ	9003	KER1	-1	
2.0	9004	KER2	10001	SCR2	BILD			-$\underline{N}$05	
3.0	9003	KER1			ADDS	12001	KTR1	P	
4.0	12001	KTR1	9002	ARC1	CHOL	12002	ATC1	510	$\underline{N}_4$
4.1			11001	KUR1	CONT				
5.0					ERRS				
5.1	11001	DUR1			CHOL	9001	RBC999	920	
5.2	9001	RBC999			INKS				
5.3					STOP				
6.0	9001	WTR1			CHOL				$\underline{N}_4$
6.1			11002	WUR1	CONT				
7.0	11002	WUR1			FLIP	12003	WLC1		
8.0	11001	KUR1	12003	WLC1	CHOL	9002	TEC1	900	
9.0	11002	WUR1	9002	TEC1	MULT	12003	DCC1		
10.0	12003	DCC1			DECO	11003	DDC1		
11.0	11003	DDC1	11004	VDC1	ROOT				-6
12.0	11004	VDC1			CODE	9002	VCC1		
13.0	11002	WUR1	9002	VCC1	CHOL	12003	DWC1	890	
14.0	12002	ATC1	12003	DWC1	ADDS	11005	AUC2		
15.0	9001	WTR1	11005	AUC2	MULT	9002	DFC1		

Fig. 1 - Facsimile of Operational Control Cards
(including normal modes)

(Ref. 19, p. 9 & 10)

FORTRAN STATEMENT

16.0		FOC1			READ			-1
17.0		FOC1	9002	DFC1	ADDS	12001	FOC2	
18.0	11001	KUR1	12001	FOC2	CHOL	9002	DTC1	910
18.1	12002	DDC1			CONT			
19.0	12002	DDC1			COLS	11006	GCC1	
20.0	12001	FOC2			FLIP		FOR2	
21.0		FOR2	12002	DDC1	MULT	11007	GCC2	
22.0	9001	WTR1	9002	DTC1	MULT	12003	FOC3	
23.0	12003	FOC3			FLIP		FOR3	
24.0		FOR3	9002	DTC1	MULT	11008	GCC3	
25.0	11006	GCC1			DECO	12002	DDC1	3
26.0	12002	DDC1			INKS			305
27.0	9002	DTC1			FILL			
28.0	10001	SCR2		DTC1	MULT	11009	RCC2	N
29.0	11009	RCC2			DECO	12001	RDC2	N
30.0	12001	RDC2			INKS			N06
31.0					HALT			

NOTE: Replace N by the number of structural elements and P by N + 1 (I6).
Replace N_4 by the number of cards of codes associated with prescribed displacements (I2).

Statement number 9, for example, means take matrix WUR1 on tape 11, position two and multiply it by matrix TEC1 on tape 9, position two and place the resulting matrix on tape 12, position 3 and call it DCC1. Instructions are performed in the sequence that they are placed in the input. Any similar type program can use the same sequence. Comment cards may be included in the pseudo instructions.

The next section of input defines the model weight at each grid point which is desired, in addition to that automatically modeled by the SAMIS program from given data. The first digits of the row and column numbers correspond to the grid point number, and the last digit gives the corresponding degree of freedom the weight is associated with, i.e. 1, 2, and 3 the x, y, and z direction respectively; 4, 5, and 6 the off center weights about the x, y, and z axis. This matrix is used to introduce concentrated loadings. Card format is as follows:

Card one

A3,I3	I6	I6,I6	I6	I6	I6	I6	I6 ...
Matrix name & no.	No. of cards of input	Matrix size	Row(-1) or column(1) listed		Precoded (1) or coded (0)		

Card two

I6	I6	I6	I6	I6	...
Column codes followed by row codes					

Card three

E12.0	E12.0	E12.0	...
Matric values row or column listed			

For coded format

Card two

I6	I6	E12.0	I6	I6	E12.0	
Row or column or column and row listed		Value or matrix element				

The next matrix data input are the nonzero accelerations associated with the restraints that are not modeled by the SAMIS program. Corresponding to the assigned loading conditions, gravity (g) accelerations are supplied in the x, y, and z direction, and (g)/in accelerations forces are supplied about the x, y, and z axis. This matrix is utilized to make equilibrium checks. Card format is the same as the weight matrix.

The next section of input gives the nonzero elements of the stiffness matrix that are not part of the elements listed. This matrix gives the holonomic boundary conditions imposed. Card format is the same as for the weight matrix.

Two cards giving the modulus of elasticity and the modulus of rigidity and a card of zeroes complete this section of input.

Tabulated grid point coordinates are then given in ascending order. If the coordinates are given in the common coordinate system, the SAMIS program makes the necessary transformations to local coordinates for generation of the various matrices. One grid point is given on each card with the following format:

I1	I3	3X,E7.0,E7.0,E7.0
Card No.	Grid point no.	Grid point coordinates

Following this section of input is the element data cards. This input describes each element as to its type, position, size, geometry and the material type it is. The card format for each of the elements is as follows:

TABLE 3

(Ref. 16, p. 114)

ELEMENT DATA FORMAT FOR FACET

CARD NO.	ELEMENT NUMBER	ELEMENT IDENTITY						
1			Elastic Node No. 1	Elastic Node No. 2	Elastic Node No. 3		Substitute Node No. 1	Substitute Node No. 2
2			Continuity Node No. 1	Continuity Node No.]	Continuity Node No. 3		Pressure	Material Temperature
3			NOT	USED	WITH	FACET		
4			NOT	USED	WITH	FACET		
5			Coord. First Local Grid Point X_1	Y_1	Z_1	Coord. Second Local Grid Point X_2	Y_2	Z_2
6			NOT	USED	WITH	FACET		
7			Coord. First Subs. Grid Point X_1	Y_1	Z_1	Coord. Second Subs. Grid Point X_2	Y_2	Z_2
8			NOT	USED	WITH	FACET		
9			Coord. First Elastic Grid Point X_1	Y_1	Z_1	Coord. Second Elastic Grid Point X_2	Y_2	Z_2

TABLE 3 (Cont'd)

(Ref. 16, p. 114)

ELEMENT DATA FORMAT FOR FACET

CARD NO.	ELEMENT NUMBER	ELEMENT IDENTITY				
1			Substitute Node No. 3	Weight	Thickness	Material Identity
2			Temperature Change Upper Surface	Temp. Change Lower Surface		
3						
4						
5			Coord. Third Local Grid Point X_3	Y_3	Z_3	
6						
7			Coord. Third Subs. Grid Point X_3	Y_3	Z_3	Coord. Identity
8						
9			Coord. Third Elastic Grid Point X_3	Y_3	Z_3	Coord. Identity

TABLE 4

(Ref. 16, p. 115)

ELEMENT DATA FORMAT FOR BEAM

CARD NO.	ELEMENT NUMBER	ELEMENT IDENTITY						
1			Elastic Node No. 1	Elastic Node No. 2	Elastic Node No. 3	Cross-Sectional Area	Polar Moment of I	Shear Area Y Force
2			Continuity Node No. 1	Continuity Node No. 2		Weight	Pressure	Material Temperature
3			NOT	USED	WITH	BEAM		
4			NOT	USED	WITH	BEAM		
5			Coord. First Local Grid Point X_1	Y_1	Z_1	Coord. Second Local Grid Point X_1	Y_1	Z_1
6			NOT	USED	WITH	BEAM		
7			Substitute Node No. 1	Substitute Node No. 2		Coord. First Substitute Grid Point X_1	Y_1	Z_1
8			NOT	USED	WITH	BEAM		
9			Coord. First Elastic Grid Point X_1	Y_1	Z_1	Coord. Second Elastic Grid Point X_1	Y_1	Z_1

TABLE 4 (Cont'd)

(Ref. 16, p. 115)

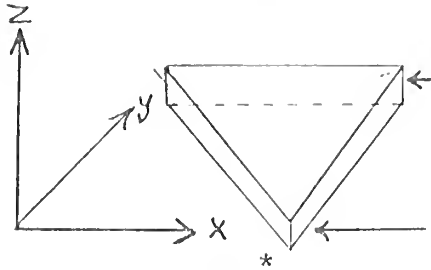
ELEMENT DATA FORMAT FOR BEAM

CARD NO.	ELEMENT NUMBER	ELEMENT IDENTITY				
1			Moment of I About Z Axis	Shear Area Z Force	Moment of I About Y Axis	Material Identity
2			Temperature Change From Zero. Str.	Temperature Gradient X-Y Direction	Temperature Gradient X-Z Direction	
3						
4						
5			Coord. Third Local Grid Point X_3	Y_3	Z_3	
6						
7			Coord. Second Substitute Grid Point X_2	Y_2	Z_2	Coord. Identity
8						
9			Coord. Third Elastic Grid Point X_3	Y_3	Z_3	Coord. Identity

TABLE 5

(Ref. 16, pgs. 124, 125, 126 & 127)

FACET ELEMENT DATA



Card	Columns	Format	Item*	Interpretation
1	1	I1	1	Indicates card number = 1.
1	2-4	I3	K	Element number. $0 < K \leq 999$.
1	5-7	1X,I2	31	Specifies the Facet element assumptions. The first digit, 3, indicates three grid points are required.
1	8-14	E7.0	GA	First, second, and third elastic grid point numbers for the element. If negative, solution displacements are produced in the local coordinate system at the grid point.
	15-21	E7.0	GB	
	22-28	E7.0	GC	
1	29-35	E7.0	--	Data in this field will be ignored.
1	36-42	E7.0	SA	First, second, and third substitute grid point numbers. If the grid point numbers are negative, solution displacements are produced in the local coordinate system at the grid point.
	43-49		SB	
	50-56		SC	
1	57-63	E7.0	M	Facet mass $M \leq 0$, total mass: $\#sec^2/in$ $M \geq 0$, mass per unit area: $\#sec^2/in^3$
1	64-70	E7.0	T	Facet thickness in inches.
1	71-72	A2	N	Name of the structural material. The first two characters of the material name must match the first two characters of the material name in the material table.

*Numbers given in this column are to be taken literally. The user must substitute appropriate numbers for letters given.

TABLE 5 (Cont'd)

FACET ELEMENT DATA

Card	Columns	Format	Item*	Interpretation
2	1	I1	2	Indicates card number = 2.
2	2-4	I3	K	Element number. $0 \leq K \leq 999$. (may be omitted)
2	5-7	1X,I2	31	Same as Card 1, columns 5-7. (may be omitted)
2	8-14	E7.0	CA	Continuity boundary conditions at the first, second, and third grid points. If both substitute points are given, these apply to the substitute points. If only elastic appear, they apply to the elastic.
	15-21	E7.0	CB	
	22-28	E7.0	CC	
2	29-35	E7.0	--	Ignored
2	36-42	E7.0	P	Normal pressure: pounds/inch^2 Positive in the plus z direction.
2	43-49	E7.0	T_m	Temperature (Degrees Rankine) of the material (used to define elastic constants).
2	50-56	E7.0	T_u	Upper surface temperature change (Degrees Rankine) from zero stress temperature.
2	57-63	E7.0	T_1	Lower surface temperature change (Degrees Rankine) from zero stress temperatuer.
2	64-70	E7.0	---	Ignored
2	71-72	A2	---	Ignored
3	Omit			
4	Omit			

*Numbers given in this column are to be taken literally. The user must substitute appropriate numbers for letters given.

TABLE 5 (Cont'd)

FACET ELEMENT DATA

Card	Columns	Format	Item*	Interpretation
5	1	I1	5	Indicates card number = 5.
5	2-4	I3	K	Element number. $0 \leq K \leq 999$. (may be omitted)
5	5-7	1X,I2	31	Same as Card 1, columns 5-7. (may be omitted)
5	8-70	9E7.0		Coordinates (inches) in the overall system of the origin of the local coordinate system (x_1,y_1,z_1); a point on the local x axis (x_2,y_2,z_2); and a point in the x-y plane (x_3,y_3, z_3), noncollinear with the first two points are selected to define desired direction of the local z axis.
5	71-72	A2	--	Ignored
6	Omit			
7	1	I1	7	Indicates card number = 7.
7	2-4	I3	K	Element number. $0 \leq K \leq 999$. (may be omitted)
7	5-7	1X,I2	31	Same as Card 1, columns 5-7. (may be omitted)
7	8-19	9E7.0		Coordinates (inches) of the substitute grid points corresponding to the first, second, and third elastic grid points.
7	71-72	A2	C	Coordinate identification. If C = L, substitute grid point coordinates are in the local system. If $C \neq L$, coordinates are overall.
8	Omit			

*Numbers given in this column are to be taken literally. The user must substitute appropriate numbers for letters given.

TABLE 5 (Cont'd)

FACET ELEMENT DATA

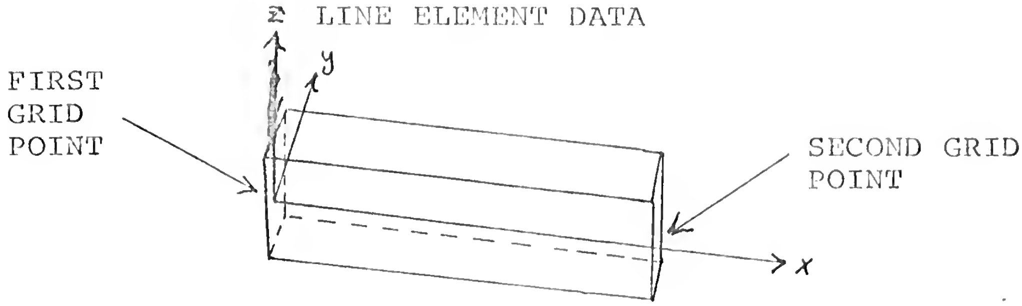
Card	Columns	Format	Item*	Interpretation
9	1	I1	9	Card number = 9.
9	2-4	I3	K	Element number. $0 \leq K \leq 999$.
9	5-7	1X,I2	31	Same as Card 1, columns 5-7. (may be omitted)
9	8-70	9E7.0		Coordinates (inches) for the first (x_1,y_1,z_1), second (x_2, y_2,z_2) and third (x_3,y_3,z_3) elastic grid points of the Facet in the overall or local system.
9	71-72	A2	C	Coordinate identification. If C = L elastic grid point coordinates are in the local system. If $C \neq L$, coordinates are overall.

*Numbers given in this column are to be taken literally. The user must substitute appropriate numbers for letters given.

TABLE 6

(Ref. 16, pgs. 128, 129, 130, & 131)

LINE ELEMENT DATA



Card	Columns	Format	Item*	Interpretation
1	1	I1	1	Indicates card number = 1.
1	2-4	I3	K	Element number. $0 < K \leq 999$.
1	5-7	1X,I2	2A	Specifies the line element equations. Rod and tube equations and (A = 1) elementary beam or (A = 2) shear beam displacement assumptions.
1	8-14	E7.0	GA	First, second, and third grid point numbers for the element. The grid number of the third grid point may be omitted. If numbers are negative, solution displacements are in the local coordinate system at the grid point.
	15-21	E7.0	GB	
	22-28	E7.0	GC	
1	29-35	E7.0	A_x	Cross-sectional area (axial) in the y-z plane: inches^2.
1	36-42	E7.0	J_x	Torsional rigidity against twist about the x axis: inches^4.
1	43-49	E7.0	A_y	Effective area deforming in shear in the x-y plane due to a y force: inches^2.
1	50-56	E7.0	I_z	Moment of inertia resisting a moment about the z axis: inches^4.

*Numbers given in this column are to be taken literally. The user must substitute appropriate numbers for letters.

LINE ELEMENT DATA

Card	Columns	Format	Item*	Interpretation
1	57-63	E7.0	A_z	Effective shear area for a shear force in the z direction: inches^2.
1	64-70	E7.0	I_y	Moment of inertia resisting a twist about the y axis: inches^4.
1	71-72	A2	N	Name of the structural material. The first two characters of the name must match the first two characters of the material table material name.
2	1	I1	2	Card number = 2.
2	2-4	I3	K	Element number. $0 \leq K \leq 999$.
2	5-7	1X,I2	2A	Same as Card 1, columns 5-7. (may be omitted)
2	8-21	2E7.0	CA CB	Continuity boundary conditions at the first and second grid points. If substitute grid points are given, these apply to the substitute points. If only elastic appear, they apply to the elastic.
2	22-28	E7.0	--	Ignored
2	29-35	E7.0	M	Line element mass. If $M \leq 0$, is the total mass (#sec^{2}in). If $M \geq 0$, M is the mass per unit length (#sec^2/in^2).
2	36-42	E7.0	P	Normal pressure: pounds/inch (positive in the plus z direction).
2	43-49	E7.0	T_m	Temperature (degrees Rankine) of the material (used to define elastic constants).
2	50-56	E7.0	T_o	Mean temperature change (degrees Rankine per unit of cross-sectional area - A_x) of the element from the zero stress temperature.

*Numbers given in this column are to be taken literally. The user must substitute appropriate numbers for letters.

TABLE 6 (Cont'd)

LINE ELEMENT DATA

Card	Columns	Format	Item*	Interpretation
2	57-63	E7.0	T_y	Temperature gradient (degrees Rankine per unit cross-sectional moment of inertia I_y) in the z direction.
2	64-70	E7.0	T_z	Temperature gradient (degrees Rankine per unit cross-sectional moment of inertia I_y) in the y direction.
2	71-72	A_2	--	Ignored
3	Omit			
4	Omit			
5	1	I1	5	Indicates card number = 5.
5	2-4	I3	K	Element number. $0 \leq K \leq 999$. (may be omitted)
5	5-7	1X,I2	2A	Same as Card 1, columns 5-7. (may be omitted)
5	8-70	9E7.0	--	Coordinates (inches) in the overall system of the origin of the local coordinate system, (x_1,y_1,z_1); a point on the local x axis (x_2,y_2,z_2); and a point in the x-y plane (x_3,y_3,z_3), noncollinear with the first two points and located to define desired plus z direction.
5	71-72	A2	--	Ignored
6	Omit			
7	1	I1	7	Card number = 7.
7	2-4	I3	K	Element number. $0 \leq K \leq 999$.
7	5-7	1X,I2	2A	Same as Card 1, columns 507. (may be omitted)

*Numbers given in this column are to be taken literally. The user must substitute appropriate numbers for letters.

TABLE 6 (Cont'd)

LINE ELEMENT DATA

Card	Columns	Format	Item*	Interpretation
7	8-21	2E7.0	SA SB	First and second substitute grid point numbers. If the grid point numbers are negative, solution displacements are produced in the local coordinate system at the grid point.
7	22-28	E7.0	--	Ignored
7	29-70	6E7.0	--	Coordinates (inches) of the substitute grid points corresponding to the first and second elastic grid points: all in the local or overall system.
7	71-72	A2	C	Coordinate identity. If C = L, substitute grid point coordinates are in the local system. If C $\neq$ L, coordinates are overall.
8	Omit			
9	1	I9	9	Card number = 9.
9	2-4	I3	K	Element number. $0 \leq K \leq 999$. (may be omitted)
9	5-7	1X,I2	2A	Same as Card 1, columns 5-7. (may be omitted)
9	8-70	9E7.0	--	Coordinates (inches) for the first (x_1,y_1,z_1), and second (x_2,y_2,z_2) elastic grid points of the line element and the grid point (x_3,y_3,z_3) defining the principal plane of the cross-section: all in the overall or local systems.
9	71-72	A2	C	Coordinate identification. If C = L, elastic grid point coordinates are in the local system. If C $\neq$ L, coordinates are in the overall system.

*Numbers given in this column are to be taken literally. The user must substitute appropriate numbers for letters.

For each material used in the structure two cards of material data are required. These cards give the material identification, the temperature associated with the material properties, the coefficient of material expansion and material stiffness coefficients. The material stiffness coefficients are the coefficients of the stress-strain equations in accordance with the following equations:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{zz} \\ \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \begin{bmatrix} D_{11} & & & & & \\ D_{21} & D_{22} & & & & \\ D_{31} & D_{32} & D_{33} & & & \\ D_{41} & D_{42} & D_{43} & D_{44} & & \\ & & & & D_{55} & \\ & & & & D_{65} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \\ \epsilon_{zz} \\ \epsilon_{xz} \\ \epsilon_{yz} \end{Bmatrix}$$

The above equations hold true for a monotropic material. If $D_{31} = D_{32} = D_{43} = D_{65} = 0$ an orthotropic material is described. An isotropic material is described if in addition $D_{33} = D_{55} = D_{66} = D_{11} = D_{41} = D_{42}$ and $D_{21} = (\nu/(1 + \nu)(D_{11})$. Then $D_{55} = E/(2(1 + \nu))$ and $D_{11} = E(1 - \nu)/((1 + \nu)(1 - 2\nu))$. Card format is as follows:

TABLE 7

(Ref. 16, p. 111)

MATERIAL TABLES INPUT DATA

Card	Field	Format	
1	1	(2X,A2,A4)	The material identification: Each material must have a unique identification number. It is recommended that standard SAE and Aluminum Association numbers be used insofar as possible. Only the leading two characters of the six-character identification number are significant.
1	2	(E8.0)	Rankin temperature for the given material properties.
1	3	(E8.0)	Coefficient of thermal expanison: inches/inch-degrees Rankin
1	4-9	(6E8.0)	Material stiffness coefficients; D_{11}, D_{21}, D_{22}, D_{31} D_{32}, D_{33}, lbs/sq.in.
2	2-8	(7E8.0)	Material stiffness coefficients; D_{41}, D_{42}, D_{43}, D_{44}, D_{55}, D_{65}, D_{66}, lbs/sq.in.

SAMPLE MATERIAL TABLE INPUT

(Isotropic Aluminum)

FORMAT	(2X,A2,A4)	(E8.0)	(E8.0)	(E8.0)	(E8.0)	(E8.0)	(E8.0)	(E8.0)	(E8.0)
CARD 1	2014T6	.53E+3	.125E-4	16.E+6	8.E+6	16.E+6	0	0	4.E+6
CARD 2		8.E+6	8.E+6	0	16.E+6	4.E+6	0	4.E+6	0
CARD 3*	00000000	00000000	00000000	00000000	00000000	00000000	00000000	00000000	00000000

*Used to indicate that all pairs of material table cards, for all materials, have been used.

The material tables must be followed by one card of zeroes to signal the end of the material tables.

The next input is the nonzero boundary conditions applied to displacements or loads. Displacements are given in inches for the restrained degrees of freedom in the x, y, and z directions and in radians for the restrained degrees of freedom about the x, y, and z axis. Loads applied in the x, y, and z directions are given in pounds and those applied about the x, y, and z axis are given in inch-pounds. The loads are those associated with the unrestrained degrees of freedom. Card format is the same as that for the weight matrix.

The last section of input is the title information cards. There are a total of eleven cards defining the displacements, generalized stiffness and generalized weight matrices. This is written as follows:

DDC 1,2,3 = DISPLACEMENT, GENERALIZED STIFFNESS AND GENERAL WEIGHT MATRICES

ROW AND COLUMN CORRESPOND TO MODE, LOAD, OR REACTION DISPLACEMENT OR EXCEPT DDC L ROW LEADING DIGITS CORRESPOND TO GRID POINT AND FINAL DIGIT TO THE DEGREE OF FREEDOM AT THE GRID POINT

REACTION NOT DISPLACEMENT AT RESTRAINT

INTERNAL FORCE FOR ELEMENT -X- (RDC X)

ROW LEADING DIGITS CORRESPONS TO GRID POINT, FINAL DIGIT

1 AXIAL FORCE	4 TORQUE
8 SHEAR ALONG X2	5 MOMENT ABOUT X2
7 SHEAR ALONG X3	6 MOMENT ABOUT X3

COL CORRESPOND TO MODE, LOAD, OR REACT. DISP. OR ACC.

The complete computer program is available in the Naval Architecture and Marine Engineering Library.

MODELING OF THE STRUCTURE

The structure analysis of a system as complicated as an icebreaker bow requires considerable preparation in order to use the SAMIS program. The first requirement is to develop a three-dimensional picture of the structure and orient it in relation to the common coordinate system. In order to clearly indicate the necessary details and label the elements and grid points, the scale must be quite large. In Appendix A, the three-dimensional drawing of the ice-breaker WESTWIND is in a scale of 1/2" = 1'. Once the three-dimensional table has been drawn, a material table must be set up to give the necessary material parameters required for the input data. Appendix B gives the material table for the icebreaker WESTWIND.

Since a ship's structure is symmetrical about the centerline, only one half of the ship need be drawn and analyzed. The boundary conditions for the nodes on the centerline can be completely described by displacements. Unless actual whipping of the ship occurs, all nodes can be fixed in the athwartship direction. Movement in the fore and aft direction will be sufficiently modeled by the facets edge, and movement in the vertical direction can be modeled as very stiff spring.

The boundary conditions of the hull are simply the average pressure loads for the depth the facet is beneath the waterline. At the bow, shell pressure loads for the ice contact area can be selected as desired. The total area in contact with the ice is that which will give the load response determined by Dr. White (Ref. 5). Shear and bending moment diagrams developed from the loading conditions imposed can be used to give the boundary forces where the structure is cut off. These can be applied as concentrated loads in the matrix data as was described as Matrix WTR1 in the previous section. This will necessitate some hand calculations to determine the section modulus and shear area. Furthermore, it would be necessary to determine the deacceleration at the time of maximum bow load and apply that to the mass of the remainder of the hull to get a deacceleration force. A check is needed to determine if these total forces satisfy equilibrium for the entire system being analyzed. A simpler method would be to assign large stiffnesses to springs connected to fixed points and supply these stiffness as matrix data input as was described by matrix KER1 in the previous section.

It would be desirable to divide the entire section being analyzed into small facets and model each stiffener as a beam. However, due to the limit on the number of elements the program can handle (999), this type of modeling can only

be done in those areas where complete and accurate detail is desired. The IIT Research Institute Technology Center of Chicago, Illinois in IITRI Project J6127 (Ref. 19), developed what is called an orthotropic plate model for stiffened structures so that large facets may be used for the remainder of the structure. Thickness is modeled as satisfying the following equation.

$$t^2 = \frac{12\ I}{A_f + A_p}$$

where

A_p = bh (the plate area

A_f = the area of the frame

$$I = A_f d^2 \frac{A_p}{A_p + A_f} + I_p + I_f$$

d = distance between the centroids of the plating and frame.

I_f = Moment of inertia of frame alone about its own centroidal axis.

$$I_p = \frac{bh^3}{12\ (1-\nu^2)}$$

b = distance between frames

h = plate thickness.

The modulus of elasticity in the plane of the plate and in the direction the stiffeners run is

$$E_y = \frac{E(A_f + A_p)}{bt(1-\nu^2 \frac{I_p}{I})}$$

The modulus of elasticity in the plane of the plate and perpendicular to the plate is

$$E_x = \frac{I_p}{I} E_y$$

and

$$E_{xy} = \nu E_x$$

The resulting material constants are then

$$D_{11} = E_y + E_{xy}^2/E_y$$

$$D_{21} = E_{xy} + E_{xy}^2/E_y$$

$$D_{22} = E_x + E_{xy}^2/E_y$$

$$D_{31} = 0$$

$$D_{32} = 0$$

$$D_{33} = G$$

$$D_{41} = E_{xy}$$

$$D_{42} = E_{xy}$$

$$D_{43} = 0$$

$$D_{44} = E_y$$

$$D_{55} = G$$

$$D_{65} = 0$$

$$D_{66} = G$$

where $G = \dfrac{E_x \; E_y}{2(1 + \nu)}$

Utilizing the SAMIS program, version one IITRI analyzed a section of the WESTWIND hull from frame seven to thirty-one. Results were not compared with stress data accumulated by the WESTWIND, but showed a structural weakness at frame 25 which has been an area of structural failures in the past.

FUTURE DEVELOPMENTS

The capabilities of this program provide the ships structure analysts with a very valuable tool. Modeling the entire shell of a ship as an unstiffened plate, and then designing the stiffeners to be most effective from the results of the first run can lead to optimal utilization of material in ship construction not only for tough skinned vessels such as an icebreaker, but also for all other types of vessels. At last there is a glimmer of hope that we can depart from the long standing method of using something that worked before in a new ship so we can have hope of some measure of success.

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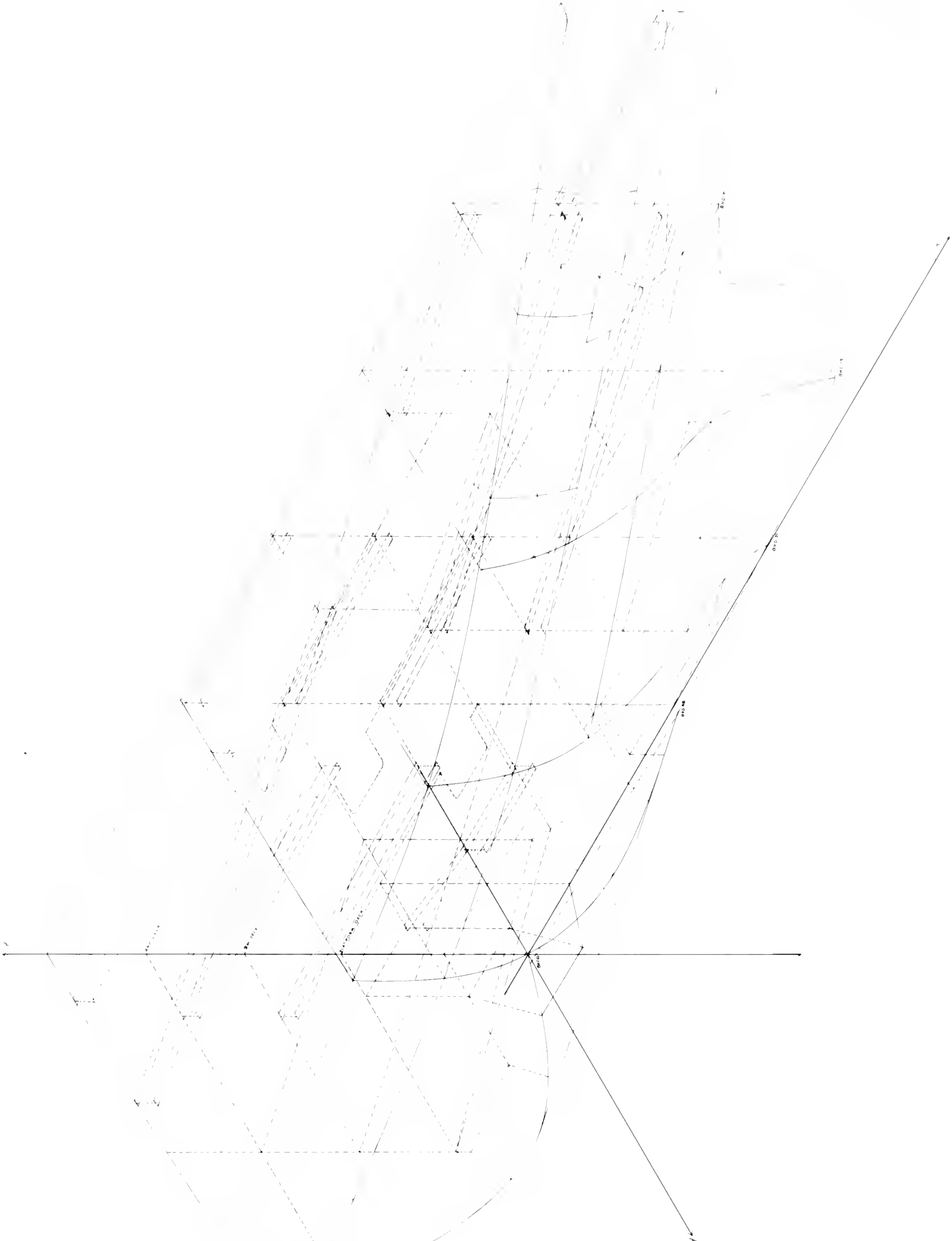


APPENDIX B

MATERIAL TABLES

(All material is medium black steel unless otherwise indicated)

<u>Shell Plating</u>

Main deck to second deck.

20.4# high strength steel plate.

Second deck to platform deck.

66.3# high strength steel plate.

Platform deck to keel.

56.1# high strength steel plate.

<u>Transverse Frames</u>

Stem, Main deck to 30° bow.

15" x 15.3# plate.

30° bow, frame 7 to keel.

40" x 20.4# plate.

30° bow, frame 7 to keel.

20" x 40.8# plate.

Cant frame 02 to frame 61, main deck to second deck.

6" x 4" x 8.25# T

Second deck to keel.

15" x 4" x 15.3# T

Note special framing detail in Appendix C.

Transverse Bulkheads

(All stiffeners on 24" centers)

Bulkhead 7

Plating

7.65# plate,

Stiffeners

6" x 4" x 11# T

Bulkhead 19

Plating

Main deck to second deck.

7.65# plate.

Second deck to platform deck.

12.75# plate.

Stiffeners

6" x 4" x 8.25# T

Bulkhead 31

Main deck to second deck

7.65# plate.

Second deck to platform deck.

10.2# plate.

Platform deck to keel.

12.75# plate.

Stiffeners

Main deck to second deck.

6" x 4" x 11# T.

Bulkhead 43

Plating

Main deck to third deck

7.65# plate.

Third deck to keel.

10.2# plate.

Stiffeners

Main deck to second deck.

5" x 2.69" x 4.48# T

Second deck to shell.

6" x 4" x 11# T

Bulkhead 61

Plating

Main deck to third deck.

7.65# plate.

Third deck to keel

10.2# plate.

Stiffeners

Main deck to second deck.

5" x 2.69" x 4.48# T

Second deck to shell

6" x 4" x 11# T

Longitudinal Bulkheads

Frame 31 to Frame 61.

Main deck to second deck.

10.2# plate

Second deck to shell.

12.75# plate

Stiffeners

Frame 31 to frame 61 on 16" centers

6" x 4" x 11# T.

<u>Decks</u>

Main deck

Plating

Stem to frame 28

10.2# plate.

Frame 28 to frame 61.

7.65# plate.

Transverse Stiffeners

Stem to frame 61

6" x 4" x 8.25# T.

Frame 34, from centerline to 10' on either side of centerline.

12" x 4" x 16.5# T

Longitudinal stiffeners

Frame 7 to frame 19, 1' off centerline at frame 7 and 5' off centerline at frame 19.

12" x 6 1/2" x 25# I

Frame 31 to frame 61, 9' off centerline at frame 31 and 19' off centerline at frame 61.

12" x 6 1/2" x 25# I

Second deck

Plating

Bow to frame 31

7.65# plate.

Frame 31 to frame 61

10.2# plate.

Deck Stiffeners. All

6" x 4" x 11# T.

Transverse beam frame 35

12" x 4" x 25# T.

Longitudinal beams

Bow to frame 7 on centerline.

12" x 6 1/2" x 25# I

1' from centerline at frame 7 to 6.0 from centerline at frame 27.

12" x 6 1/2" x 25# I.

Frame 31 to frame 43 on centerline.

12" x 6 1/2" x 25# I.

Frame 43 to frame 61, 6.0' from centerline.

12 x 6 1/2" x 25# I

Third deck

Plating (All)

10.2# plate.

Transverse stiffeners

Bow to frame 61

7" x 6 3/4" x 15# T.

Frame 45 and frame 49 from centerline to 6' on either side of centerline.

12" x 6 3/4" x 30# T

Longitudinal stiffeners

1' from centerline at frame 7 to 9' from centerline at frame 31.

Frame 31 to frame 43 on centerline.

Frame 43 to frame 61, 6' from centerline.

16" x 7" x 36# I

12.75# brackets, 21" x 10 1/2", used to tie into frame at end of each deck stringer.

Platform Deck

Plating

Frame 7 to frame 31 and frame 43 to frame 61.

10.2# plate.

Transverse stiffeners (All)

7" x 6 3/4" x 30# T.

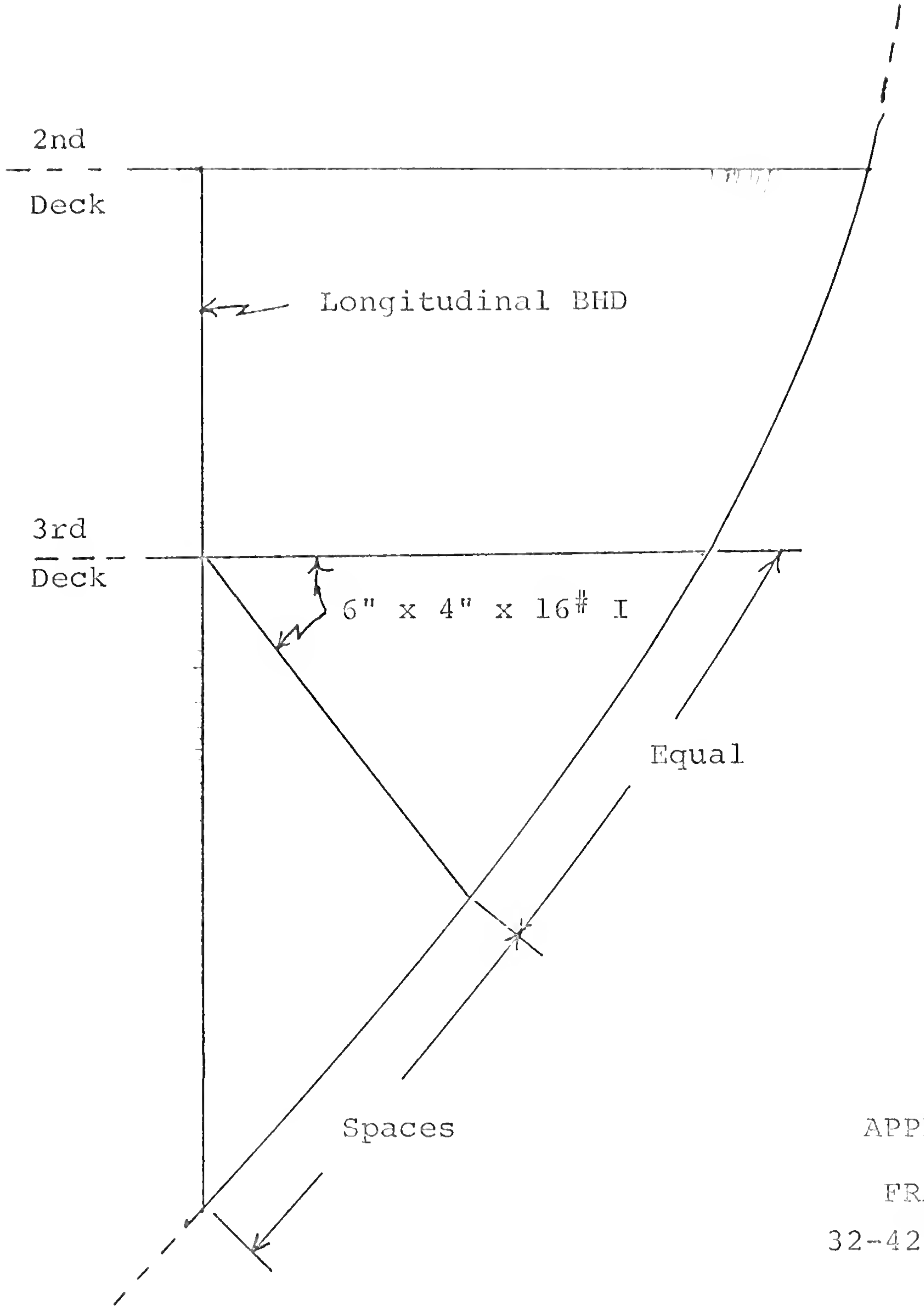
Frame 44 and frame 48, from centerline to 6' on either side of centerline.

12" x 6 3/4" x 30# T.

Longitudinal stiffeners

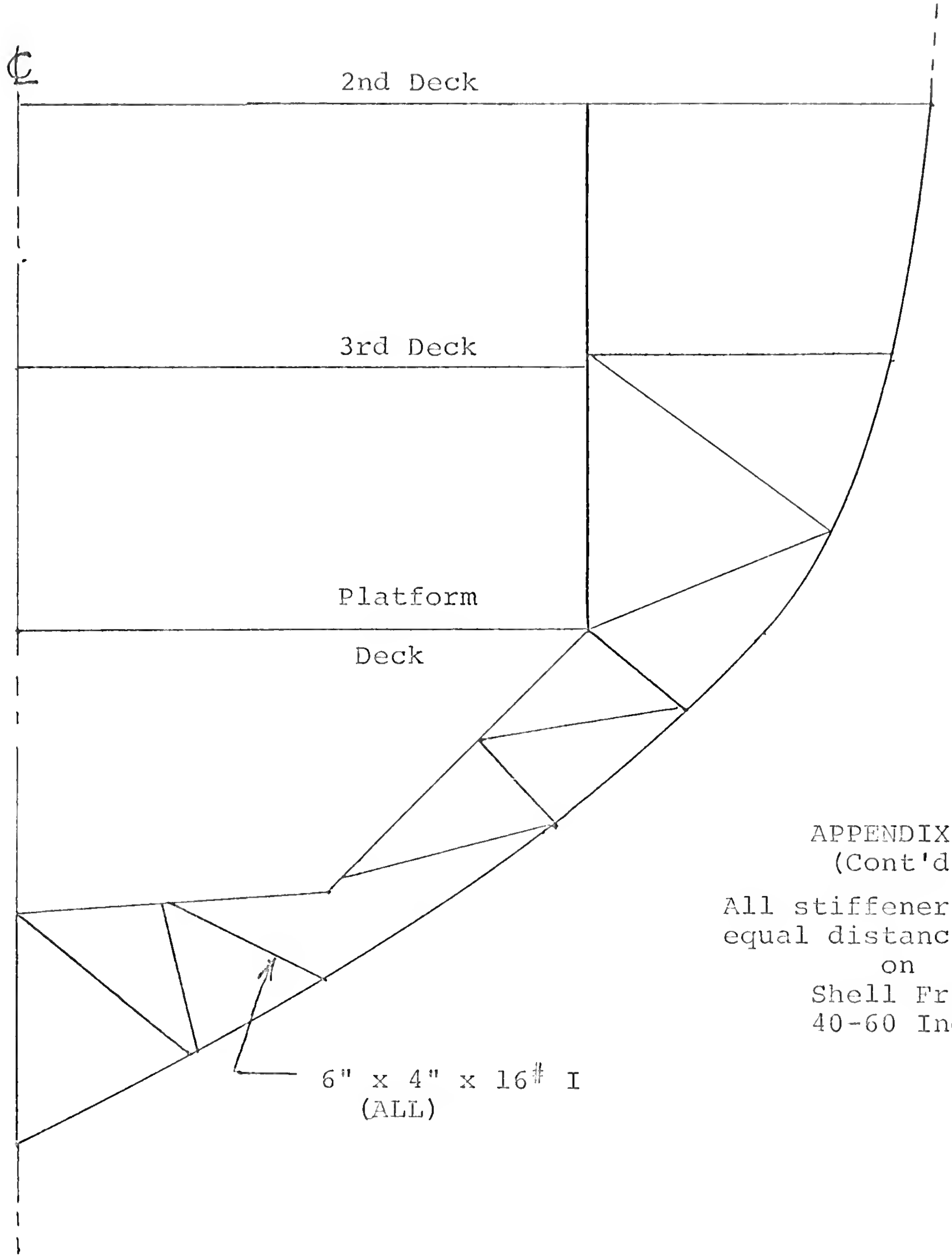
Frame 43 to frame 56 on centerline.

16" x 7" x 36# I.



APPENDIX C

FRAMES
32-42 Incl.



APPENDIX C
(Cont'd)

All stiffeners spaced
equal distance apart
on
Shell Frames
40-60 Incl.